

Math 112 review questions for the second midterm

The midterm covers everything we have done in class, tutorial and hand-in homework, up to Monday November 7th. I will give you a sheet of the order axioms O1 - O4 if there is a question where you need to prove something using the axioms.

- (1) Make a list of the important topics that we have covered so far, and the types of questions that you have seen done in class and in homework. Make a list of the important theorems and make sure that you understand what each theorem means.
- (2) Define the following terms (and be able to give an example if appropriate): Binary relation, inequality, order, well-ordered, transitive, divides, greatest common divisor, factor, relatively prime, prime, twin primes, the division algorithm, the Euclidean algorithm, Diophantine equation.
- (3) What does the “well-ordering axiom for \mathbb{Z} ” say? What does the “well ordering theorem” say and why was it controversial?
- (4) Give an example of a set with an order that is well-ordered, and another example of a set with an order that is not.
- (5) Prove using only the order axioms that for any numbers a , b and c , if $a < b$ then $a + c < b + c$.
- (6) Can the numbers in one’s digit arithmetic have an order on them that satisfies the axioms? What about the numbers in mod 5 arithmetic? What about mod n ?
- (7) State two unsolved conjectures about prime numbers.
- (8) Pick your two favorite two- or three-digit numbers, factor them into a product of primes, and compute their greatest common divisor using your factoring and then again using the Euclidean algorithm.
- (9) Suppose a is a number such that for any other number b , either $(a, b) = 1$, $(a, b) = 0$ or $(a, b) = a$. Must a be a prime number?
- (10) State the fundamental theorem of arithmetic.
- (11) Do practice problems 2.5 to 2.8, 3.1 to 3.8, and 4.1, 4.3 and 4.4. There are solutions to these at the end of the chapters so you can check your work.
- (12) These questions from the Exercises at the end of each chapter are also good practice. Those marked with * were homework.
 - 2.10*, 2.11, 2.14*, 2.15, 2.17*
 - 3.2* (note: on the midterm, you will not have any problems that require a calculator), 3.3, 3.7*, 3.8*, 3.9*, 3.10*, 3.11, 3.12*,
 - 4.1, 4.2*, 4.4b and d*, 4.6*, 4.18
- (13) Use the Euclidean algorithm to find a solution to the diophantine equation $17x + 5y = 1$ Now that you know one solution, how do you find another one? How do you find infinitely many solutions and what do they look like?