Homomorphisms between Diffeomorphism Groups

Kathryn Mann

University of Chicago

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- $\mathrm{Diff}_c^r(M)$ = group of compactly supported C^r diffeomorphisms isotopic to the identity.
- This is a simple group [Mather, Thurston], so any nontrivial homomorphism is necessarily injective.

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“Induced” means $\Phi(g) = fgf^{-1}$

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Question (Ghys, 1991)

Let M_1 and M_2 be closed manifolds.
$\exists$ *(injective) homomorphism* $\mathrm{Diff}^\infty(M_1)_0 \hookrightarrow \mathrm{Diff}^\infty(M_2)_0$

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- Can also ask this for general boundaryless M and $\mathrm{Diff}^r_c(M)$.

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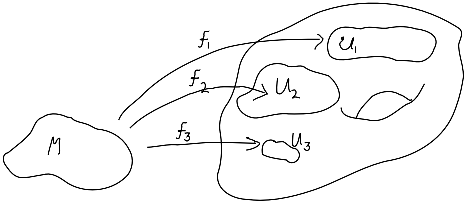

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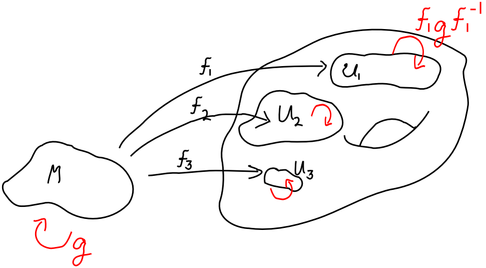

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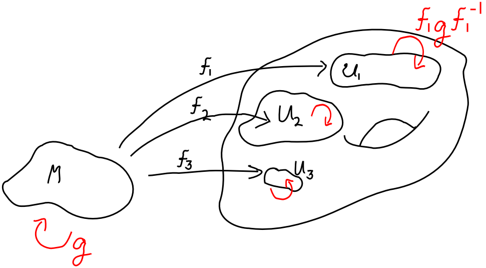

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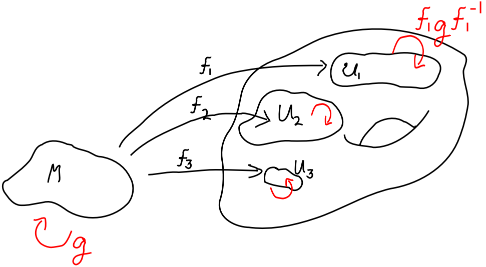

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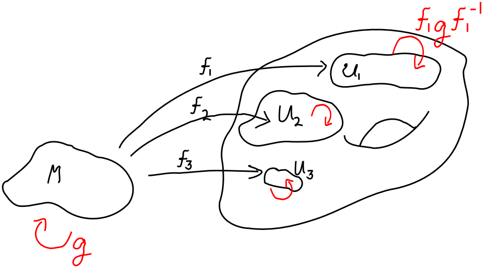

- Special cases: $M_2 = M_1 \times N$, unit tangent bundle of M_1...

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Can this happen with Diff instead of $\mathbb{R}$?
How bad can injections $\mathrm{Diff}_c^r(M_1) \to \mathrm{Diff}_c^p(M_2)$ look?

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Let $r \geq 3$, $p \geq 2$; M_1 *and* M_2 *1-manifolds.*

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(This answers Ghys' question in the $\dim(M_2) = 1$ case)

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- Build f_i

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Open set $U \leftrightarrow G^U$ group of diffeomorphisms fixing U pointwise.

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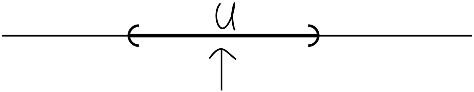

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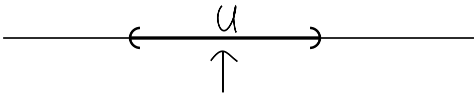

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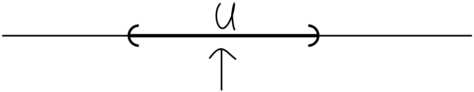

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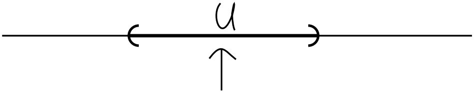

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These also [mostly] work for S^1, but not for general M!

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For U, V open subsets of $\mathbb{R}$

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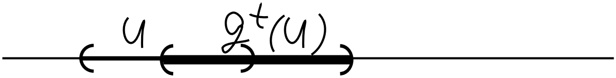

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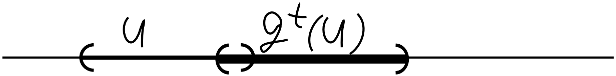

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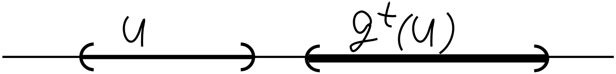

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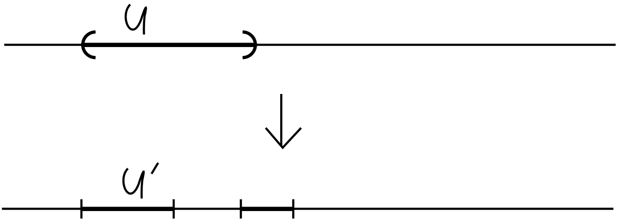

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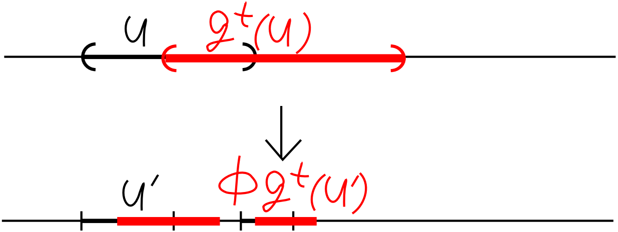

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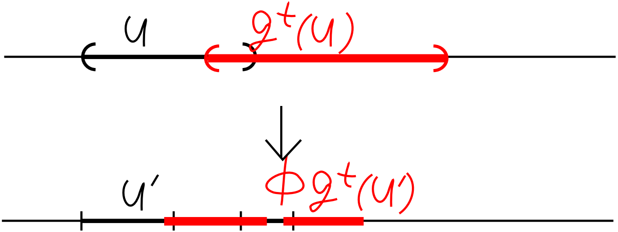

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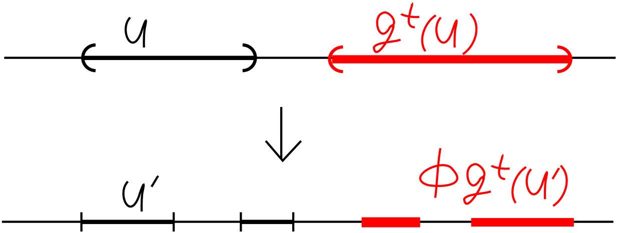

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Replacing one or both $\mathbb{R}$'s with S^1 isn't too hard.

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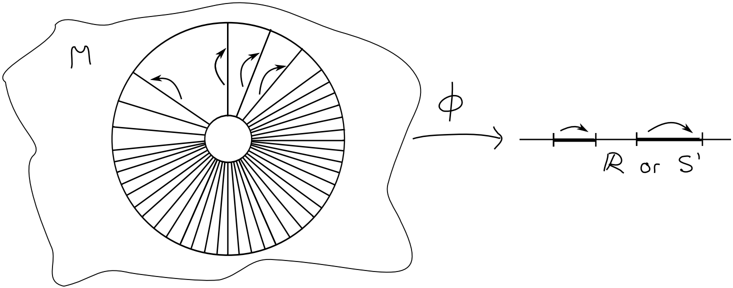

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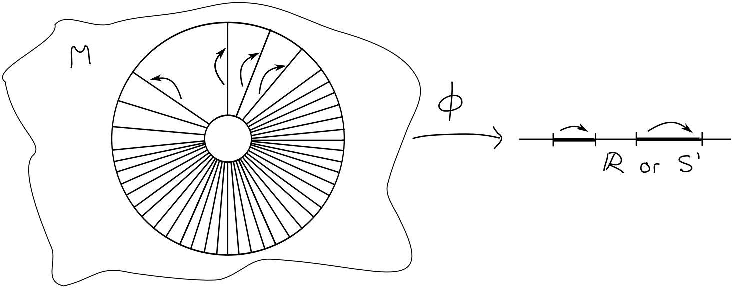

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Answering Ghys' question

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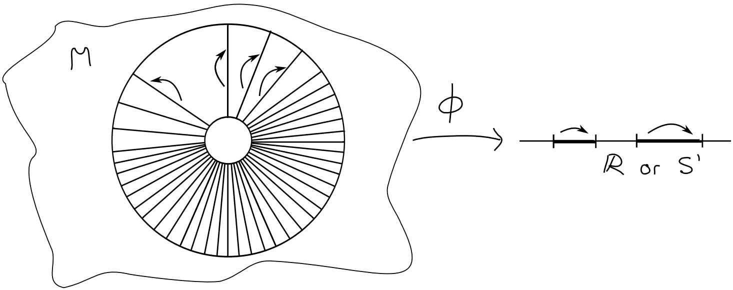

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$\mathrm{Diff}^2_c(\mathbb{R})$ has the property that each element has the same centralizer as its square. Not so in $\mathrm{Diff}^2_c(M)$.

Things you can think about

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