Math 113 Homework 8. Due 11/3

Reading corresponding to lectures:
Tuesday 10/27 DF 7.3
Thursday 10/29 DF 7.4
Tuesday 11/3 DF 7.5

Problems to hand in:

1. Do the following problems from DF section 7.3: 1, 2, 6, 8, 9, 18a, 22
2. Use the "reduction homomorphism" trick to prove that $x^2 + 3y^2 = 102$ has no integer solutions $x, y \in \mathbb{Z}$. (hint: mod 4 is always a good place to start).
3. Define $\phi : \mathbb{C} \to M_2(\mathbb{R})$ by $\phi(a + bi) = \left(\begin{smallmatrix} a & -b \\ b & a \end{smallmatrix}\right)$. Show that ϕ is a ring homomorphism. What is its kernel and image? What subring of $M_2(\mathbb{R})$ can you now say is isomorphic to $\mathbb{C}$?

   Not to hand in: Can you find an analogous homomorphism from the quaternions to $M_2(\mathbb{C})$? To $M_4(\mathbb{R})$?

4. Prove that the set of matrices with all entries zero except for the first column is a one-sided (but not two-sided) ideal of $M_n(\mathbb{R})$. For example in $M_2(\mathbb{R})$, these are the matrices of the form $\left(\begin{smallmatrix} a & 0 \\ b & 0 \end{smallmatrix}\right)$.
5. Do the following problems from DF section 7.4: 8, 10, 14a,b, 15 a (use problem 14),
6. Imitating the proof from class (done on Thursday), show that the ideal generated by x^2 and $4x$ in $\mathbb{Z}[x]$ is not a principal ideal. Is it a principal ideal in $\mathbb{Q}[x]$?