Problems to hand in:

1. Do the following problems from DF section 7.3: 1, 2, 6, 8, 9, 18a, 22

2. Use the “reduction homomorphism” trick to prove that \( x^2 + 3y^2 = 102 \) has no integer solutions \( x, y \in \mathbb{Z} \). (hint: mod 4 is always a good place to start).

3. Define \( \phi : \mathbb{C} \to M_2(\mathbb{R}) \) by \( \phi(a + bi) = \begin{pmatrix} a & -b \\ b & a \end{pmatrix} \). Show that \( \phi \) is a ring homomorphism. What is its kernel and image? What subring of \( M_2(\mathbb{R}) \) can you now say is isomorphic to \( \mathbb{C} \)?

   Not to hand in: Can you find an analogous homomorphism from the quaternions to \( M_2(\mathbb{C}) \)? To \( M_4(\mathbb{R}) \)?

4. Prove that the set of matrices with all entries zero except for the first column is a one-sided (but not two-sided) ideal of \( M_n(\mathbb{R}) \). For example in \( M_2(\mathbb{R}) \), these are the matrices of the form \( \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} \).

5. Do the following problems from DF section 7.4: 8, 10, 14a,b, 15 a (use problem 14),

6. Imitating the proof from class (done on Thursday), show that the ideal generated by \( x^2 \) and \( 4x \) in \( \mathbb{Z}[x] \) is not a principal ideal. Is it a principal ideal in \( \mathbb{Q}[x] \)?