Math 113 Homework 2. Due 9/15

Reading corresponding to lectures:
Tuesday 9/8: DF 1.2, 1.6
Thursday 9/10 DF 1.6, 2.1. We will never cover 1.4 or 1.5, and will return to 1.7 later.
Tuesday 9/15 2.3, 2.4, (possibly 2.5 if we have time)

Problems to hand in:

1. Do these problems from DF Section 1.2: 2, 8, 9, 10.

2. Do these problems from DF Section 1.6 (you should wait until after class on Thursday): 1, 2, 4, 6, 13.

3. Do the following problems from DF section 2.1
   2, 4 (hint: you can use the group $(\mathbb{Z}, +)$).

4. Let $\phi : G \to H$ be a homomorphism, and assume that the image of ϕ is H (in other words, ϕ is *surjective*). Suppose that the set S generates G. Show that $\phi(S)$ generates H.

5. *Braids and Permutations:*
   In class, we defined a homomorphism from the braid group to the symmetric group as follows: Given a braid on n strands, label the points at the top and bottom by numbers $1, 2, 3, ...n$. Following the strands from bottom to top defines a bijective function σ from the set $\{1, 2, ...n\}$ to itself. Precisely, $\sigma(k)$ is the label on the top end of the strand that starts from the bottom point labeled k.

   a) Let $a_1, a_2, ..., a_n$ be the "basic braids" defined in the previous problem set. What element of the symmetric group corresponds to $a_1 a_3 a_4^{-1} a_2^{-1} a_1$?
   (write its cycle decomposition).
   b) What is the inverse of the element of the symmetric group from part a? (use the cycle decomposition)
   c) What element of the symmetric group corresponds to the inverse of $a_1 a_3 a_4^{-1} a_2^{-1} a_1$?
   d) Now prove in general, that if $\phi : G \to H$ is a group homomorphism, then $\phi(g^{-1}) = (\phi(g))^{-1}$.
   e) Give examples of 3 different braids on 3 strands that correspond to the identity element of the symmetric group.

6. A *transposition* in S_n is a cycle of length two consisting of numbers that differ by 1. For example, (12) and (56) are transpositions.
   Use problem 4 and the fact that the braid group is generated by $\{a_1, a_2, ...a_{n-1}\}$ that the set of all transpositions generates S_n.

7. *Multiplication tables:*
In this exercise, we prove that S_3 has "the same" multiplication table as D_6.
(a) Complete the multiplication table for S_3. (As usual, we use the word "multiplication" to mean "binary operation for the group". So this table tells you what happens when you compose two permutations.)

$\circ$	e	(123)	(132)	(12)	(13)	(23)
e	e	$e \circ (123) = (123)$				
(123)	$(123) \circ e = (123)$	$(123) \circ (123) = (132)$				
(132)	$(132) \circ e = (132)$					
(12)						
(13)						$(13) \circ (23) = (132)$
(23)						

(b) Complete the multiplication table for D_6 below:

$\cdot$	e	r	r^2	s	sr^2	sr
e	e	$er = r$				
r	$r \cdot e = r$	$r \cdot r = r^2$				
r^2	$r^2 \cdot e = r^2$					
s						
sr^2						$sr^2 \cdot sr = r^2$
sr						

(c) In what sense are these multiplication tables "the same"? Can you phrase this in terms of a *isomorphism* between the two groups?

(d) Notice that each element appears exactly once in each column and once in each row of the tables. Prove that this is true for the multiplication table of any group. (hint: "equations have unique solutions" from the second lecture)