Math 113 Homework 1. Due 9/8

Reading corresponding to lectures:

- Tuesday 9/2: DF 1.1 and 1.3
  Thursday 9/4: DF 1.3, 1.2,
  Tuesday 9/9: DF 1.2 (more), 1.6, beginning of chapter 2.
- General: read the first two sections ("introduction" and "basic properties") of the wikipedia article on the braid group. `http://en.wikipedia.org/wiki/Braid_group`

Problems to hand in:

1. Do these problems from DF Section 1.1:
   7, 17, 20, 22, 25, 26.

2. Recall that a group is called *abelian* if all pairs of elements commute, i.e. $ab = ba$ for all a and b in the group. Prove that the braid group B_n is not abelian whenever $n \geq 3$.

3. In class, we defined basic braids a_1, a_2, ... by saying that a_i is the braid with a single crossing, where the i^{th} strand crosses *over* the adjacent $i+1^{th}$ strand. Draw the 5-string braid
$$a_1 a_3 a_4^{-1} a_2^{-1} a_1$$

4. What is the inverse of the braid $a_1 a_3 a_4^{-1} a_2^{-1} a_1$? Write it as a product of the a_i's, and then draw it.

5. Do the following problems from DF section 1.3. I recommend you do problem 1 of this section for practice, and check your work. You do not need to hand in problem 1.
   4a., 8 (hint: one way is to show that S_ω has at least n elements, for any integer n),
   9b and c, 15, 16.

6. (a) Prove that the group of positive rational numbers with operation multiplication, denoted $(\mathbb{Q}^{>0}, \times)$ is generated by $\{1/p \mid p \text{ is prime }\}$.

   (b) Consider the group $(\mathbb{Q}^{>0}, \times)$ of positive rational numbers, with operation $\times$ (usual multiplication). Can it be generated by any *finite* set? Find a finite generating set, or explain why one does not exist.

7. In this problem, we'll see that finite-order elements can generate an infinite group.
   Let $a = \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}$ and $b = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.
   These are matrices in $GL_2(\mathbb{R})$. Show that $|a| = 6$, $|b| = 4$ but that infinitely many matrices in $GL_2(\mathbb{R})$ can be written as products of a and b and their inverses.

Note regarding question 7: the set of all matrices that can be written as products of a and b and their inverses is called the *subgroup generated by a and b.*

For a different (but related) example, see page 64 in DF