Math 113 Review for midterm

Material covered
You are responsible for all of the material covered in class, on homework (problem sets), and in the following sections of the textbook:

Sections covered on the first test: 0.3, 1.1, 1.2, 1.3, 1.6, 1.7, 2.1-2.4, 2.5 (not all of 2.5)
Sections covered since the first test (in the order we did them): 4.1, 4.2, 3.1-3.3, 4.4, 4.5.

Vocabulary list
I expect you to know all of the vocabulary on the **review for test 1**, especially that concerning group actions. Also, be able to define and give an example of the following additional terms

> Fiber, left coset, right coset, quotient group, $|G : H|$ the index of H in G, representative for a coset, HK (the "mega coset"), automorphism, inner automorphism, Sylow p-subgroup.

Practice problems
Make sure you know how to do all of the problems on the homework

Make sure you can do all of the problems from the **review sheet for test 1**

New problems from DF:

- Section 4.1, # 1, 2, 10
  (in 1 and 2, G_a denotes the stabilizer of a, which we also called "stab(a)"
- Section 4.2 #3a. This is an equivalent statement of the problem:
  List the elements of D_8 as $1, r, r^2, r^3, s, sr, sr^2, sr^3$ and label these with integers $1, 2, \ldots, 8$. Cayley's theorem says that D_8 is isomorphic to a subgroup of S_8, and gives a homomorphism $\phi : D_8 \to S_8$ where $\phi(g)$ is the permutation of the elements of D_8 that is given by left-multiplication by g, according to our labeling. For example, $\phi(r) = (1234)(5876)$ since 1 corresponds to the identity, and $r \cdot e = r$, which corresponds to 2, so $\phi(r)$ is a permutation that sends 1 to 2, etc.
  Write the permutation corresponding to each element of D_8.
- Section 3.1 # 3, 5 (you can use the fact from problem 4 – we know this), 10, 11 (use $\mathbb{R}$ instead of F), 13 (compare to problem 12 that you did in homework), 17a-d, 20, 22a, 24, 29, 34.
- Section 3.2 # 1, 2 (don't worry about what a "proof" should look like, just convince yourself that everything is correct), 15, 16
- Section 4.4 #1 (we did this in class), 3 (hint: look at orders of elements), 6, 7 (use the definition of "characteristic" from the book).
- Section 4.5 # 1, 8, 13.
- Imitating our proof from class for Z_5, show that $Aut(Z_p) \cong Z_{p-1}$ for any prime p.
- Imitating our proof from class, show that every group of order 21 is cyclic.
- Show that every group of order 12 contains a normal subgroup (we may do this example in class, if so, don't cheat and look at your notes!)

Theorem statements
A copy of the following page will be attached to your midterm. You may refer to it when writing your midterm.

Cayley's Theorem
Let G be a finite group of order n. Then G is isomorphic to a subgroup of the symmetric group S_n.

Lagrange's Theorem
Let H be a subgroup of G. Then $|H|$ divides $|G|$, and the number of left cosets of H in g is $|G|/|H|$.

The four isomorphism theorems

1. Let $\phi : G \to H$ be a homomorphism. Then $\ker(\phi) \trianglelefteq G$ and $G/\ker(\phi) \cong \phi(G)$

2. Let G be a group, H, K subgroups of G and assume $H \leq N_G(K)$. Then HK is a subgroup of G, $K \trianglelefteq HK$, $H \cap K \trianglelefteq H$ and
$$HK/K \cong H/(H \cap K)$$

3. Let G be a group and H and K normal subgroups of G, with $H \leq K$. Then $K/H \trianglelefteq G/H$ and
$$(G/H)/(K/H) \cong G/K$$

4. (partial statement) Let G be a group and $N \trianglelefteq G$. There is a bijection from the set of subgroups of G that contain N to the set of subgroups of G/N.

Sylow's theorem
Let G be a group of order $p^\alpha m$, where p is a prime and not a factor of m.

- G has a Sylow p-subgroup (a subgroup of order p^α).
- Any two Sylow p-subgroups are conjugate
- Let n_p denote the number of Sylow p-subgroups. Then $n_p = |G : N_G(P)|$ for any Sylow p-subgroup P, and $n_p \equiv 1 \pmod{p}$.

Useful Definitions

Normalizer: $N_G(H) = \{g \in G \mid gHg^{-1} = H\}$

Centralizer: $C_G(A) = \{g \in G \mid \text{ for all } a \in A,\ gag^{-1} = a\}$

Center: $Z(G) = \{g \in G \mid \text{ for all } x \in G,\ gxg^{-1} = x\}$