Math 112 homework #6. Due 11/7

The mathematician is in much more direct contact with reality [than the physicist]. This may seem a paradox, since it is the physicist who deals with the subject-matter usually described as 'real'; but a very little reflection is enough to show that the physicist's reality, whatever it may be, has few or none of the attributes which common sense ascribes instinctively to reality. A chair may be a collection of whirling electrons, or an idea in the mind of God: each of these accounts of it may have its merits, but neither conforms at all closely to the suggestions of common sense.

A mathematician, on the other had, is working with his own mathematical reality [...] 317 is prime, not because we think so, or because our minds are shaped in one way rather than another, but because it is so, because mathematical reality is built that way

– G. H. Hardy

(1) Review up to the end of Chapter 3, and read Chapter 4. You may skip the section in Chapter 4 on "division algorithm with a negative dividend"

(2) Do exercises 1 and 2 from the Prime Numbers reading.

(3) In class we showed that if you assume there are only finitely many primes $p1, p2, ..., pk$ and you build the number $(p1 \times p2 \times ... \times pk) + 1$, you get a contradiction. Work through the details of the argument to show that you get a contradiction if you assume the only prime numbers are 2, 3, 5, and 7. What number N do you build? Why cant it be divisible by 2 or 3 or 5 or 7? What is its prime factorization?

(4) Use the Euclidean algorithm to compute the following Greatest Common Divisors:
   (a) (654,321)
   (b) (999999,2)
   (c) (143,91)
   (d) (48,36)

(5) In class on Wednesday, we stated the following fact: *if a and b are two integers, and there are two other integers x and y such that $ax + by = 1$, then (a,b)=1.*

   Prove this fact. Hint: let $d = (a, b)$. Does d divide 1?

(6) In tutorial, you discussed how to use the Euclidean algorithm to solve *Diophantine equations*. Use the Euclidean algorithm to find integers x and y such that $72x + 35y = 1$.

(7) Do exercises 3.3, 4.2, and 4.4c) and d) from the course notes.