Math 113 homework due 1/23

Two elements are needed to form a truth: a fact and an abstraction.
– Remy de Gourmont

(1) Read and review Chapter 11 in the book and the handout on the classification of finite simple groups.

(2) For this question we will work in D_{10}, the group of symmetries of the pentagon. The standard form for these symmetries that we established in class is given in the following list:

$$I, R, R^2, R^3, R^4, F, FR, FR^2, FR^3, FR^4$$

Use the relations $R^5 = I$, $F^2 = I$, and $RF = FR^4$ to write each of the following in one of the standard forms.
(a) FRF, (b) $(R^2)(R^4)(R^3)$, (c) $(FR)(R^2)(FR)(FR)$, (d) $RFRF$

(3) Show that D_8, the group of symmetries of the square, is generated by R_{90} (rotation by 90 degrees)and F (vertical flip).

(4) Recall that Z_6 is the group of rotations of the hexagon. It has elements I, R, R^2, R^3, R^4, R^5 and the single relation $R^6 = I$. The element R clearly generates this group (meaning every element is obtained by combining R with itself).
(a) Does R^2 generate the group?
(b) Does R^3 generate the group?
(c) Does R^5 generate the group?
In each case, explain why or why not.

(5) Question 11.9 Hint: there is an easy way to do this!

From the reading on the classification of finite simple groups:

(6) Show that there is no subgroup of D_8 that is structurally the same as C_3.

(7) According to Ewes, what are the roles of abstraction and classification in mathematics? Why are they important? Do you agree?

(8) What do you think is the benefit (or lack thereof) of a mathematical proof that no one understands? You may cite some of Ewes' opinions if you like, but should add your own as well. Speculate on this proof in particular – do you think it is useful to know of a group that has 808,017,424,794,512,875,886, 459,904,961,710,757,005,754,368,000,000,000 elements?