Math 113 homework due 1/18

(note that 1/16 is a holiday)

> *"...what a wealth, what a grandeur of thought may spring from what slight beginnings."*
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> – H. F. Baker (1866-1956), British mathematician, on the concept of a *group*

(1) Read and review Chapter 11 in the book

(2) a) How many ways can you position the numbers 1, 2, 3 and 4 on the vertices of a square? (a different number on each vertex). Make a complete list or explain how you accounted for all of them.
b) We saw in class on Wednesday that for the triangle every configuration of the numbers 1, 2 and 3 was the result of applying a symmetry to the triangle which was labeled 1, 2, 3 in counterclockwise order starting from the top. Is the same true for the square? If not, which configurations *can't* come from symmetries and why?
c) Are there configurations of 1, 2, 3, 4, 5 on the pentagon that can't come from taking a pentagon with the counterclockwise-from-the-top order and applying a symmetry? If so, describe them.

(3) Exercise 11.1

(4) a) Write a multiplication table for the rotations of the square.
b) Is the set of rotations of the square a *group* (and hence a *subgroup* of the group of symmetries of the square)? Why or why not? (explain why it satisfies all the conditions of being a group, or show which one(s) it does not satisfy)
c) What about the reflections of the square together with the identity?
d) Are the rotations of a regular n-gon always a subgroup of symmetries of the n-gon? Explain.

(5) Show that for any regular polygon, two flips always make a rotation. (hint: one way is to look at the order of labeling of vertices)

(6) Since two flips always make a rotation, what is the *angle* of the rotation? What information do you need to know about the lines of symmetry of the flips in order to answer this question? Try some experimentation to help you discover the answer.

(7) Problem 11.8 (Two groups are *structurally the same* if their multiplication tables look the same, but some elements have been given different names. Example: the table in 11.1a is for a group that is structurally the same as the symmetries of the triangle)