Math 141 Review for the Midterm

The midterm covers all the content of the course so far (up to GP 2.2) with an emphasis on more recent material. I do not expect you to answer any difficult problems using the material from 2.2, just to have read it. Much like on the 30-minute test, you are expected to be able to give precise definitions, to use Theorems in order to solve problems and give short proofs. The questions will be easier than most homework problems, but some problems will be longer than those on the 30-minute test. I will ask some true/false or multiple choice questions to test your understanding of theorem statements or examples.

Here are some strategies to help you review.

1. Make a list of important words, concepts, and theorems. Give definitions and precise theorem statements, and give an example (or non-example) related to each.
2. Look at the solutions to the 30-minute test and make sure you understand how to solve the problems.
3. Review the past homework (including HW4, the review for the 30 minute test). Expect to see test questions closely related to a homework problem, or to the additional problems below.

Practice problems by topic

1. Basic topology: GP 1.1 #7, 11.
2. Manifolds and tangent spaces: GP 1.1. #17 and 1.2 #1, 7, 8, 10.
3. Inverse function theorem, local immersions, submersions...: GP 1.3 #2 and 1.4 #1.
   What are the critical and values of the map $f : \mathbb{R}^3 \to \mathbb{R}^2$ given by $f(x, y, z) = (x^2y^2, y^2z^2)$?
4. Transversality, homotopy and stability:
   GP 1.5 #2 (you did this before, but it's good practice), 6, 7.
   Let X be the open 2-dimensional disc, $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$. Show any two maps of X into the torus $S^1 \times S^1$ are homotopic.
   Show that if X and Y are contractible manifolds, then the product manifold $X \times Y$ is also contractible.
   Summarize the content of the stability theorem. What does it show?
5. Sard's theorem and measure zero:
   Suppose that $X_1, X_2, ...X_j$ are smooth manifolds, and for each i, f_i is a smooth function from X_i to a manifold Y. Let $S = \{y \in Y \mid y \text{ is a critical value for some } f_i\}$. Is S necessarily measure zero? Prove or give a counterexample.
6. Tangent bundles and Whitney: GP 1.8 #5.
   If $f : X \to Y$ is a submersion, is $df : T(X) \to T(Y)$ also a submersion? What about the same question with "immersion" instead of submersion? With "diffeomorphism"?
7. Manifolds with boundary: GP 2.1 #2, 3, 4, 8, 10 (hint for 10: use charts!)