

Math 141 Homework 9

Reading for this week:

GP section 2.3. This is a difficult section, take your time and draw pictures as you read to help illustrate the proofs.

Problems to hand in

1. Let $S^1 \subset \mathbb{R}^2$ be the standard unit circle. Prove that $N(S^1)$ is diffeomorphic to $S^1 \times \mathbb{R}$.
2. Let $X = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = 1, z = 0\}$ (this is also a circle). Show that $N(X)$ is diffeomorphic to $S^1 \times \mathbb{R}^2$.
3. (a) Let $Y = S^2 \subset \mathbb{R}^3$ be the unit radius sphere. Let $\epsilon = 1/2$. Write an explicit formula for a submersion (as in the ϵ -neighborhood theorem) $\pi : Y^\epsilon \rightarrow Y$, that sends each point in Y^ϵ to the “closest point” in Y . Prove that your function is a submersion.
(b) For which values of ϵ is it impossible to do part a)? What goes wrong?
4. Do the following problems from GP section 2.3 #4, 7, 10, 13