Math 141 Homework 8

Reading for this week:
I encourage you to read ahead to section 2.3, which we will discuss next week.
On Thursday after the midterm, we discussed the classification of 1-manifolds, given in the appendix. You may wish to read this, but you are not required to know this material for the final exam. There is another (just for fun!) interesting article on the classification of 1-manifolds on the course website.

Problems to hand in

1. Do the following problem from GP section 2.1 #8, (this was on the review for the midterm)

2. Let X be the unit ball in $\mathbb{R}^N$. Let $\vec{n} : \partial X \to \mathbb{R}^N$ be the function from the previous problem. Describe $\vec{n}$.

3. Now we look at a similar construction for submanifolds instead of boundaries. Suppose X is a k-dimensional manifold in $\mathbb{R}^n$ (you may assume X has no boundary). Suppose $Y \subset X$ is a l-dimensional submanifold. We know that $T_y(Y) \subset T_y(X)$ for every point $y \in Y$.
   (a) Define the *normal space to* Y *at* y to be the set
   $$N_y(Y) = \{v \in T_y(X) : v \text{ is orthogonal to } T_y(Y)\}.$$
   Show that the dimension of $N_y(Y)$ is the codimension of Y.

   (b) Define the *normal bundle to* Y to be the set
   $$\{(y, v) \in Y \times \mathbb{R}^N : v \in N_y(Y)\}$$
   Show that the normal bundle is a manifold (of what dimension?)