# Math 141 Homework 6

**Reading for this week:**
GP section1.7, up to but not including the Hessian and the Morse lemma
GP beginning of section 1.8. – we will continue with 1.8 next week.
See also extra notes on measure zero on website.

**Not to hand in** Review the solutions to the test. If you struggled with it, I recommend that you try re-writing it on your own time, to test if you have since learned the material.

**Problems to hand in**

1. Do problem 8 from GP section 1.6. (finishing the proof of the stability theorem, which was rushed in class).

   Not to hand in: I recommend you also write, for yourself, a summary of the proof of all the parts of this theorem and how they are related. You'll notice many parallel techniques.

2. Measure zero sets:
   (a) Suppose that $A_1, A_2, A_3, ...$ are measure zero sets. Show that the union
   $$\bigcup_{i=1}^{\infty} A_i$$
   is also measure zero.
   (b) Consider $\mathbb{R}^2 \subset \mathbb{R}^3$ as the set of points of the form $(x_1, x_2, 0)$. Show that $\mathbb{R}^2$ has measure zero in $\mathbb{R}^3$. (hint: use part a) to see that it is enough to show that a set of the form $\{(x_1, x_2, 0) \mid n \leq x_1 \leq n+1,\ m \leq x_2 \leq m+1\}$ has measure zero.
   (c) Generalize part $b$ to show that $\mathbb{R}^k$ has measure zero in $\mathbb{R}^n$, for $k < n$.

3. Do the following problems from GP section 1.7:
   3 (hint: use part c from the problem above!), 2, 6.

4. Do problem 1 from GP section 1.8.