Math 141 Homework 4: *Not to hand in.*

Reading for this week: GP sections 1.4 and 1.5. Review past sections.
The 30 minute test on Tuesday will cover GP sections 1.1 - 1.5. You will not be asked to prove anything using material from 1.5, but will be expected to know the definitions there. You should have a basic understanding of the topics in topology covered the first week of class: connectedness, compactness, continuity.

Review for your test:

1. Make a list of important words and concepts. Define and give an example of each.
2. Make a list of the main theorems that we have proved. Give an example related to each theorem – for instance, for the local submersion theorem, give a simple example of a local submersion at a point, and show how it is equivalent to the canonical one. Then give an example of something that is not a local submersion. Aim to make your examples as simple as possible, and use pictures to illustrate.
3. For more basic practice with tangent spaces: try exercises #2, 3 and 4 in GP section 1.2.
4. Show that $SL(2, \mathbb{R})$ is a submanifold of $M_2(\mathbb{R})$ by showing that 1 is a regular value of the determinant function $\det : M_2(\mathbb{R}) \to \mathbb{R}$.
5. To check your understanding of the concept of *transversality* from 1.5, do problem 2 parts (a) to (f) in 2.5. The answers (with no justification) are on the next page.
6. Examples: Give an example of a smooth map that is not a diffeomorphism. Give an example of an injective, smooth map that is not an immersion. Give and example of an immersion that is not an embedding. Explain why the cone (figure 1-2 on page 2) is not a manifold.

Challenge problems, optional
These problems cover more material on matrix groups, which were mentioned briefly in class. They are interesting, but completely optional and not needed for the test. If you're interested in learning more about matrix groups as manifolds, there is a really nice book called "Matrix Groups for Undergraduates" by K. Trapp. Highly recommended, and easier to read than GP.

1. Show that the set of 2×2 orthogonal matrices is a submanifold by direct computation. Try to generalize to $n \times n$. Now look at the proof on page 22 of GP using directional derivatives. Note that sB is the matrix B with all entries multiplied by s.
2. Show that matrix multiplication is a smooth map from $O(n) \times O(n)$ to $O(n)$. Try the $O(2)$ case first if you prefer.
3. Now try problems 9-13 from GP 1.4. Problem 13 is quite challenging!

Answer to 1.5 #2:
the linear space that intersect transversely are a, b, d only in the case where $n \leq k + l$, and f.