Comments on HW 3

GP 1.3 #1.

General idea: If A is nonsingular, this map is bijective with inverse is $y \mapsto A^{-1}(y-b)$. If A is singular, then it has some kernel, so the map is not injective. The important thing to take away from this problem is that the derivative of this map at any point x is the linear map A. (this should help you understand the derivative of a function f as "the best linear approximation" when you pretend x is 0 and $f(x)$ is 0). When the inverse exits, its derivative is A^{-1}, and all partial higher order derivatives of both are zero, showing this is smooth with smooth inverse.

GP 1.3 #5.

You need to show that f is a bijection, smooth with smooth inverse, and that $f(X)$ is open. The trickiest part is to show that $f(X)$ is open, for this you need to appeal to the definition of *local diffeomorphism* to say that there is a neighborhood of any point $y \in f(X)$ that is also in the image $f(X)$.

GP 1.3 #6.

a) Given $f : X \to Y$ and $g : W \to Z$ recall that $f \times g$ is the map $X \times W \to Y \times Z$ given by $f \times g(x, w) = (f(x), g(w))$. Work with coordinate charts, and show that in coordinates the jacobian of $f \times g$ is a matrix of the form $\begin{pmatrix} df_x & 0 \\ 0 & dg_w \end{pmatrix}$.

b) use the chain rule

c) and d) are left to you!

GP 1.3 #9

For a), use the hint! Now define f to be projection onto the $e_{i_1}, ...e_{i_k}$ coordinate plane given by the lemma. Show that df_x is surjective, in fact bijective. This means that f is a local diffeomorphism.

b) Show that, if $\phi : U \subset X \to V \subset \mathbb{R}^k$ is the projection from part a) that gives local coordinates, then its inverse is a map of the form $a \mapsto (a, g(a))$.

Problem 2: this will be discussed in class.

GP 1.4 # 1 Use the local submersion theorem, and show that the projection of an open set in $\mathbb{R}^k$ to the first l coordinates has open image.

GP 1.4 #7 Use the hint!

Since X and Y are the same dimension, when y is a regular value the derivative is not only surjective at any point x_i in $f^{-1}(y)$ but injective, so the inverse function theorem applies, and gives a neighborhood W_i where f is a local diffeomorphism. Note that each W_j does not contain x_i when $i \neq j$. Thus, the points x_i cannot accumulate anywhere. Since X is compact, there can only be finitely many points x_i. Now you can take $V_i \subset W_i$ that are all disjoint.

Problem 4. The only critical value is 0. Hopefully you had no trouble graphing the level sets!