

Math 141 Homework 2. Due 2/2

Reading: Guillemin and Pollack (henceforth GP) sections 1.1 and 1.2

Problems to hand in:

1. Do the following problems from GP section 1.1:

4, 6, 10, 18.

Notes:

For problem 10, the manifold $S^1 \times S^1 \subset \mathbb{R}^4$ is the product manifold as defined in class (or in the book), where $S^1 \subset \mathbb{R}^2$ is as defined on Page 4 of the book. You do not need to give a proof that the $b = a$ and $b > a$ cases are not manifolds, just a sentence of explanation.

Also, problem 18 part b) has a typo, it should be $g(x) = f(x - a)f(b - x)$. This function is called a “bump function” and is indeed very useful!

2. This problem shows that there is a smooth map (though not a diffeomorphism!) from \mathbb{R} to a V shape in \mathbb{R}^2 .

Let $f(x)$ be the same function as defined in GP 1.1 problem 18. You may use the results of that problem in this one.

(a) Draw the image of the function $F : \mathbb{R} \rightarrow \mathbb{R}^2$ given by $F(t) = \begin{cases} (-f(-t), f(-t)) & \text{if } t < 0 \\ (f(t), f(t)), & \text{if } t \geq 0 \end{cases}$

(b) Show that the function F is smooth.

(c) (not to hand in) Why is F not a diffeomorphism?

3. Do the following problems from GP section 1.2:

5, 12.