Math 141 Homework 11

Reading for this week:
GP section 2.5 and 2.6

Problems to hand in

1. Do the following problems from GP section 2.5 # 1, 4, 5 (you'll find hints at the end of the section), 8 (this was done in class, but I want you to write the argument out)

2. Do the following problems from GP section 2.6 # 1 (to show two statements are equivalent, you need to show that one implies the other, and vice versa).

3. The "ham sandwich theorem" states that given three objects in $\mathbb{R}^3$ (i.e. a piece of bread, a piece of ham, and another piece of bread), there exists a single straight cut with a knife that simultaneously bisects each object into two halves of equal volume. (this is maybe not surprising if you're used to cutting nice looking sandwiches, but the theorem says that it doesn't matter if the bread and ham are floating separately in space and shaped like unicorns– you can still make an even cut.)

   More precisely: *If A_1, A_2 and A_3 are solids in $\mathbb{R}^3$, then there exists a 2-dimensional plane P such that exactly half the volume of each A_i lies on either side of P.*

   Prove the ham sandwich theorem using Borsuk-Ulam.

   (hints: Define a function $g : S^2 \to \mathbb{R}$ as follows. For each $x \in S^2$ (thought of as a unit vector in $\mathbb{R}^3$ let P_x be the plane with normal vector x. Let $v(x)$ be the volume of A_1 on the side of the plane P_x that contains x, and $w(x)$ be the volume of A_1 on the other side. Let $g(x) = v(x) - w(x)$. Of course you can define similar functions for A_2 and A_3. Now use one of the consequences of Borsuk-Ulam...)

4. Optional (not to hand in): See problem 2 in GP 2.6 for an alternative approach to the dimension 1 case of Borsuk-Ulam.