Math 141 Homework 1. Due 1/26

Reading: Hatcher notes on topology, optional additional reading on website.

Problems to hand in:

1. (not to be graded, I just want to get to know you) What math classes have you taken other than calculus?

2. The $\epsilon - \delta$ definition of continuity for functions from $\mathbb{R}^n$ to $\mathbb{R}^n$ is given below. Show that this is *equivalent to* the definition of continuity given in class (the pre-image of every open set is open). In other words, you need to show that a function is $\epsilon - \delta$ continuous if it is "pre-image open" continuous, and vice versa.

   A function $f : \mathbb{R}^n \to \mathbb{R}^n$ is continuous if, for any point x, and any ϵ, there exists δ such that $\text{dist}(x, z) < \delta$ *implies* $\text{dist}(f(x), f(z)) < \epsilon$.

3. *(if this is familiar, look at the just for fun problems below).* Here are the letters of the alphabet: ABCDEFGHIJKLMNOPQRSTUVWXYZ

   Which letters are homeomorphic to A? Which letters are homeomorphic to M? Which letters are homeomorphic to O? You do not need to show work.

4. Let $A \subset \mathbb{R}^n$ be a set. The *interior* of A is the set of points x in A such that there is an open ball $B \subset A$ with $x \in B$. The *boundary* of A, written ∂A, is the set of points $x \in \mathbb{R}^n$ such that every open ball B containing x contains both points in A and $\mathbb{R}^n \setminus A$
   (a) Give an example of a set A such that $A = \partial A$.
   (b) Prove that the boundary of the open ball $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$ is the unit circle.
   (c) Give examples of three different sets in $\mathbb{R}^2$ that all have boundary equal to the unit circle.

5. Do the following problems from the Hatcher notes, page 28: #1, #4.

6. Suppose that X and Y are topological spaces, X is compact, and $f : X \to Y$ is continuous. Prove that the image $f(X)$ of f is a compact subset of Y.

7. (a) Give an example of an open cover of $\mathbb{R}^2$ that has *no* finite subcover. Conclude that $\mathbb{R}^2$ is not compact.

   (b) Give an example of an open cover of the long strip $[0, 1] \times \mathbb{R}$ that has *no* finite subcover. Conclude that the strip is not compact.

   (c) Give an example of an open cover of the open unit ball in $\mathbb{R}^2$, i.e. the set $\{(x, y) : x^2+y^2 < 1\}$ that has *no* finite subcover. Conclude that the open ball is not compact.

8. Do problem #8 from Hatcher, page 42

Just for fun:

1. The course website has the first chapter of a book called "intuitive topology", it consists of challenging exercises to show that certain surfaces are homeomorphic. Try it! (solutions are included).

Remark: in fact, the "deformations" that you are being asked to do produce something stronger than a homeomorphism – two shapes that can be deformed one into another are called isotopic. *Can you think of an example of two homeomorphic sets that are not isotopic?*