Independent project topic suggestions

1. Design a project of your own choice! E-mail me with what you propose to do. If you want help designing the project, or finding resources, talk to me in office hours.

   Or, if you prefer, here are some suggestions to get you started:

2. Performa a critical examination of Book 1 of Euclid's Elements. Discuss the proofs (and failings or shortcomings in Euclid's proofs) of a few propositions of your choice, ideally with a common theme. Discuss what additional justification, axioms, etc. would be necessary to make the proofs rigorous. Suggested first reading: online version of Euclid and commentary at `http://aleph0.clarku.edu/~djoyce/elements/bookI/bookI.html`.

3. Proof the only-if direction of Gauss' regular constructible n-gons.
   Reference: *Conjecture & Proof* by M. Laczkovich, chapter 3.
   Alternatively, if you have experience with Galois theory (e.g. from Math 114), you can do look at Dummit and Foote ch. 14, prop 14.29 for a proof of the other direction.

4. Prove that you cannot equidecompose a tetrahedron into a cube of the same volume. See the book *Conjecture & Proof* (M. Laczkovich), or section 27 of Hartshorne (these give different proofs, both are nice).

5. Explore constructions with compass and *marked* ruler. You have already trisected the angle this way, show how to construct the regular 7-gon. What else can be done? Reference: Hartshorne chapter 30 and/or 31.

6. The Banach–Tarski paradox. Explain how to decompose a sphere, rotate and translate the pieces, and produce two spheres (or a least get as far as the concept of *paradoxical decompositions* of sets. Requires some knowledge of group theory.

7. The "Cartesian plane" with coordinates in fields other than $\mathbb{R}$. You encountered $\mathbb{Q}^2$ in a homework question, learn what happens for other fields. Can you use finite fields? Possible reference: Hartshorne Chapter 14-15. (there are many other resources)

8. What are the axioms for an *affine plane*? What are some interesting models for this geometry? Possible reference: part II of E. Moorehouse's book, available here: `http://www.uwyo.edu/moorhouse/handouts/incidence_geometry.pdf`

9. Any other chapter from Hartshorne that we didn't cover in class. A few suggestions:
   - Hilbert's axioms of incidence for circles.
   - Euclid and Hilbert on area, sections 22-23
   - Geometries based on algebra (fields!), sections 14-15.
   - Euclid's theory of volume (26)

10. Cover a topic from Stillwell's book that we didn't do in class (e.g. from chapter 6). check with me first!

11. Learn how to construct an *error correcting code* using projective geometry.
    Reference: google turned up this for me `http://micsymposium.org/mics_2009_proceedings/mics2009_submission_52.pdf`, I bet you can find more with a quick search.

12. Last summer, I wrote some course notes for a fun class called *Do-it-yourself hyperbolic geometry*. Look at them here, and pick a topic to explore further. `https://math.berkeley.edu/~kpmann/DIYhyp.pdf`.
    Sample topics:
    - Areas of circles, polygons, and triangles in hyperbolic space
    - Can you build a hyperbolic soccer ball?
    - What shapes tile the hyperbolic plane?

13. There are 17 ways of tiling the Euclidean plane with repeating patterns (see `https://en.wikipedia.org/wiki/Wallpaper_group` for pictures). Learn why! (the explanation on the wikipedia page isn't very good, I can help you find a friendlier explanation). This topic uses elementary group theory.

14. Look at the book *Experiencing geometry* by David Henderson. (I have a copy if you want to borrow it). It is wonderful to read, and there are many good project topics! One suggestion: read the first few chapters discussing "straightness", aim to learn the content of Chapter 4 or Chapter 5.