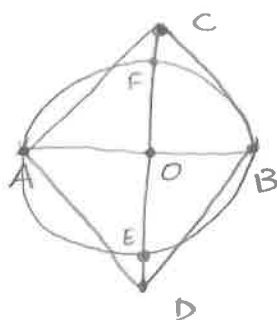# Midterm Solutions.

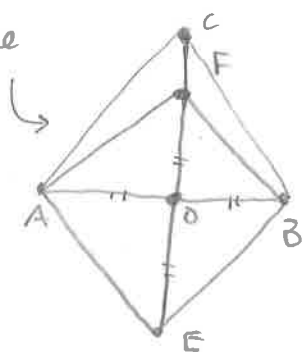**(1a.)** Given A, B construct two equilateral triangles ABC, ABD, with base AB. Form line CD, this meets AB at O.



Draw a circle centered at O with radius OB. This meets the line CD at E and F.

Form segments AF, FB, BE and EA. AFBE is a square.

**(1b.)** We have



from part 1a.

$|AO| = |OB| = |OE| = |OF|$ since all are radii of the circle.

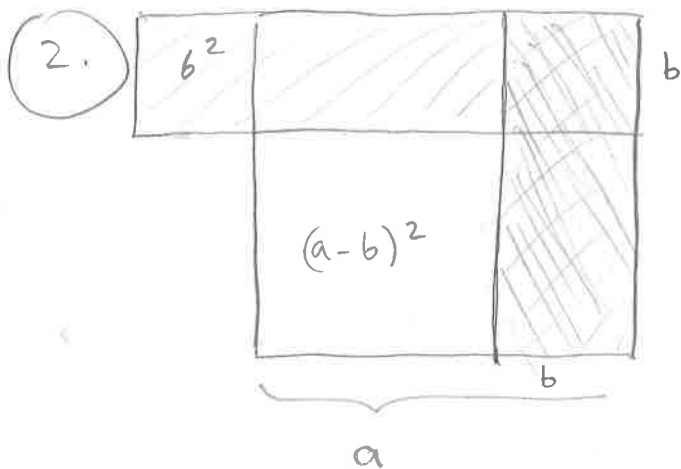Since $|CA| = |CB|$, by S-S-S$_n$ applied to triangles COB and COA, we have $\angle AOC = \angle BOC$

So both are $90°$. Similarly, $\angle AOE = \angle BOE = 90°$

Now S-A-S applied to BOE, AOE, FOA, and FOB shows! all these triangles are congruent, so all former sides of the square have the same length.

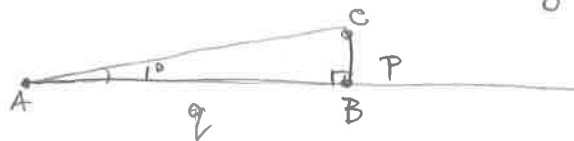Also, $\angle AEB = \angle AEO + \angle OEB = 2\angle AEO = 2 \cdot \dfrac{(180 - 90)}{2}$

$= 90°$. The same is true for all other vertices, so we have a square.

since $\triangle AOE$ is isosceles

**2.**

$b^2$ | | $b$

$(a-b)^2$

$b$

$a$

Total area is $a^2 + b^2$.
shaded regions each have
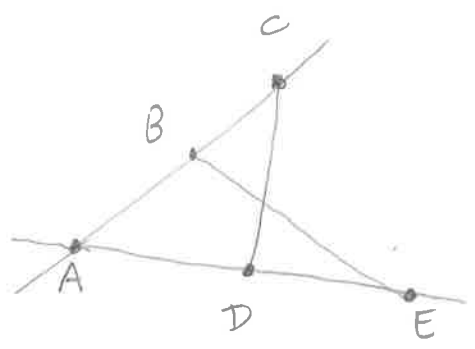area $ab$

So $a^2 + b^2 - 2ab = (a-b)^2$

**3.**

Suppose for contradiction that $\tan(1°) = \frac{p}{q}$.
~~Then~~ We can construct length $q$, a right angle
and length $p$, therefore a $1°$ ~~angle~~ angle.



By repeating this construction using $AC$ as the base,
we can then construct $2°$, and inductively any angle that
is an integer number of degrees.

But we proved in class that a $20°$ angle
is not constructable.

(4)

A, B, E are not collinear,
since A, B, C are but
E lies on a diffht line then
A and C.
Similarly A, D, C are not collinear.

Apply Pasch's axiom to triangle ABE, and line CD;
conclude that CD intersects either segment $\overline{AB}$
or segment $\overline{BE}$. If it intersected $\overline{AB}$,
then DC would have to lie on line AC
(by I1) but A, D and C are not collinear.

Thus, CD intersects $\overline{BE}$, say at X.

Apply Pasch's axiom now to ACD, and
conclude as above that, BE intersects the segment $\overline{CD}$
                          ∧
                         line
say at Y.

Since X and Y both lie on BE and CD, they must
be the same point (by I1).
So X = Y and this lies on $\overline{BE} \cap \overline{CD}$.