Math 130 Midterm

October 6, 2015

Instructions: Answer all questions. You have the full class period. You are given Hilbert's axioms on the next page, and may use that paper for scratch work.

1. (a) (5 points) It is the first day of Prof. Euclid's geometry class, and you have just learned how to use a straightedge and compass. Prof. Euclid then shows you how to construct an equilateral triangle. You are now to do the following construction, using only what you already know:

   Given two points A and B, show how to construct a square with opposite (i.e. non adjacent, as in the picture) vertices A and B.

   You do not have to prove that the construction gives a square, just give the construction.

   (b) (5 points) Now explain why your construction from part a) makes a square. You may use any high-school level geometry e.g. SAS, congruence, talking about symmetry...

2. (4 points) Draw a picture to represent the algebraic identity $(a-b)^2 = a^2 - 2ab + b^2$.
   Label or explain your picture so that I can understand it.

3. (5 points) Prove that $\tan(1^\circ)$ is irrational. You may use results from class if you state them clearly. Do not use extra results from homework.

4. (11 points) You are working in a geometry that satisfies Hilbert's axioms I1-I3 and B1-B4.

   Assume $A * B * C$, and $A * D * E$, and that the points A, C and E are not collinear.
   Prove, from the axioms only, that the segments BE and CD intersect each other. Make a drawing to illustrate the configuration of points.

   Hint: Show first that the segment BE intersects the *line* CD

Hilbert's incidence and betweenness axioms:

I1. For any two distinct points, there is a unique line containing both.

I2. Every line contains at least two points

I3. There exist three noncollinear points.

B1. If $A * B * C$, then A, B, and C lie on a line, and also $C * B * A$

B2. For any distinct points A, B, there exists C such that $A * B * C$

B3. Given A, B, C distinct points on a line, exactly one is between the other two.

B4. (Pasch's axiom)
Let A, B, C be three points that do not lie on a line, and let L be a line which does not meet any of the points A, B, C. If L contains a point between A and B, then it also contains a point between A and C or between B and C, but not both.