HW 6 Selected solutions

1. This was done in class on Tuesday 10/20

2. Suppose ℓ is a common factor of m and n.
Let $(\bar{a}, \bar{b}) \in (\mathbb{Z}/m\mathbb{Z}) \times (\mathbb{Z}/n\mathbb{Z})$

Then $(\bar{a}, \bar{b})^{\frac{mn}{\ell}} = \left(\overline{\left(\frac{mn}{\ell}\right) a}, \overline{\left(\frac{mn}{\ell}\right) b}\right)$

$= (\bar{0}, \bar{0})$ since m divides $\frac{mn}{\ell}$
and n divides $\frac{mn}{\ell}$.

So every element has order at most $\frac{mn}{\ell}$, so cannot be a generator.

3. $50 = 2 \cdot 5^2$
For $p = 2$, the Sylow 2-subgroups are all of the conjugates of $\langle s \rangle$. There are 25 of these, they are all of the form $\langle r^k s \rangle$ $k = 0, 1, \dots 24$

for $p = 5$, $n_5 \equiv 1 \mod 5$ and divide 2, so $n_5 = 1$
There is one sylow-5 subgroup of order 25
it is $\langle r \rangle$.

4. $|S_5| = 5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 3 \cdot 40$
$n_3 \equiv 1 \mod 3$ and divides 40, so is 1, 4, 10 or 40.
Any 3-cycle e.g. $\sigma = (123)$ will generate an order 3 subgroup.
There are 10 distinct 3-cycles (since $\binom{5}{3} = 10$)
So there are at least 10 sylow 3 subgroups. Since all are conjugate, these are the only ones - each must be generated by an order 3 element, i.e. a 3-cycle.