MATH 185-1: Complex Analysis

Homework #7

Due March 17, 2016

All problems are from Gamelin, *Complex Analysis*, unless stated otherwise. If you use an exercise that has not been shown on a previous assignment or in class, prove it first before applying it.

1. Exercise V.i.7
2. Exercise V.2.10
3. Exercise V.2.12
4. Exercise V.3.4
5. Exercise V.4.1
6. Exercise V.4.3
7. Exercise V.4.12
8. Exercise V.4.13

Extra Credit: Show that the uniform limit of analytics functions is analytic using the Cauchy integral formula through the following steps.

(a) Let $\{f_k(z)\}_{k=0}^{\infty}$ be a sequence of analytic functions on a disk $\{z : |z-z_0| < \rho\}$ that converges uniformly to $f(z)$ on $\{z : |z-z_0| < \rho\}$. Show that for $0 < r < \rho$,

$$f(z) = \frac{1}{2\pi i} \oint_{|z-z_0|=r} \frac{f(w)}{w-z} dw,$$

for all z such that $|z - z_0| < r$.

(b) Use part (a) to show that for $|z - z_0| < r$

$$f'(z) = \frac{1}{2\pi i} \oint_{|z-z_0|=r} \frac{f(w)}{(w-z)^2} dw.$$

(You may want to use Exercise III.1.6 from Homework #4.)

(c) Conclude that if $\{f_k(z)\}_{k=0}^{\infty}$ is a sequence of analytic functions on a domain D that converges uniformly to $f(z)$ on D, then $f(z)$ is analytic on D.