MATH 185-1: Complex Analysis

Homework #5

Due March 3, 2016

All problems are from Gamelin, *Complex Analysis*, unless stated otherwise. If you use an exercise that has not been shown on a previous assignment or in class, prove it first before applying it.

1. Let $h(z)$ be a continuous function defined on a domain D and $z_0 \in D$. Denote the average value of $h(z)$ on the circle $|z - z_0| = r$ by

$$A(r, z_0) = \frac{1}{2\pi} \int_0^{2\pi} h(z_0 + re^{i\theta}) d\theta.$$

   Show that for every $\epsilon > 0$, there exists a $\delta > 0$ such that if $r < \delta$, then $|A(r, z_0) - h(z_0)| < \epsilon$. (Note that this is a rigorous formulation of the fact that the average value of $h(z)$ on a circle centered at z_0 approaches the value of the function at the center z_0 as the radius decreases towards 0.)

2. Exercise III.3.2

3. Exercise III.4.1

4. Exercise III.5.3

5. Exercise III.5.5

6. Exercise IV.1.5

7. Evaluate $\int_\gamma e^z dz$, where γ is the straight line segment from the origin to $1 + 2i$ by (a) parametrizing the path, and (b) finding an appropriate primitive for the integrand.

8. Exercise IV.2.5