MATH 141: Differential Topology

Homework #1

Due September 11, 2014

Please turn in the starred (*) problems only.

1. Let $(X, \mathcal{T})$ be a topological space and $A \subset X$.

   (a) Show that every open set contained in A is contained in $\mathrm{Int}(A)$. Conclude that
   $$\mathrm{Int}(A) = \bigcup_{U \in \mathcal{T}, U \subset A} U.$$
   (In other words, $\mathrm{Int}(A)$ is the largest open subset contained in A.)

   (b) Show that every closed set containing A contains $\bar{A}$. Conclude that
   $$\bar{A} = \bigcap_{C \text{ is closed}, A \subset C} C.$$
   (In other words, $\bar{A}$ is the smallest closed subset containing A.)

2. Let $X = \mathbb{R}$, and let S be the set of irrational numbers. Find $\bar{S}$ in the usual topology, the trivial topology, the discrete topology, and the cocountable topology.

3.* Find a topology on $\mathbb{R}$ along with a subset $S \subset \mathbb{R}$ such that not every limit point of S is a limit of a sequence in S.

4.* Show that $\mathbb{R}^n$, with the usual topology, has a countable basis.

5. If Y is a subspace of $(X, \mathcal{T})$ and Z is a subspace of Y, show that Z is a subspace of X.

6.* Let $(\mathbb{R}^n, \mathcal{T})$ be $\mathbb{R}^n$ with the usual topology (i.e. from the Euclidean metric). Let $A \subset \mathbb{R}^n$ have the subspace topology. Show that a set $O \subset A$ is open in the subspace topology if and only if for each $x \in O$, there exists an $\epsilon > 0$ such that all points of A of distance less than ϵ lie in O.