Math 1B: Calculus **Spring 2020**

Discussion 19: Maclaurin and Taylor Series

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1 Maclaurin Series

Find the Maclaurin series for the given function.

1. $f(x) = \arctan x^2$

we know $\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$

Therefore, $\arctan x^2 = \sum_{n=0}^{\infty} (-1)^n \frac{(x^2)^{2n+1}}{2n+1} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+2}}{2n+1}$

2. $f(x) = x \cos 2x$

$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$

$x \cos 2x = \sum_{n=0}^{\infty} (-1)^n \frac{x \cdot (2x)^{2n}}{(2n)!} = \sum_{n=0}^{\infty} (-1)^n \frac{4^n x^{2n+1}}{(2n)!}$

3. $f(x) = x^2 \ln(1 + x^3)$

$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$

$x^2 \ln(1+x^3) = \sum_{n=0}^{\infty} (-1)^{n-1} x^2 \cdot \frac{(x^3)^n}{n} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{3n+2}}{n}$

2 Application of Taylor Series

Use the series to evaluate the limit.

1. $\lim_{x\to 0} \frac{x-\ln(1+x)}{x^2}$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$$

$$\lim_{x\to 0} \frac{x-\ln(1+x)}{x^2} = \lim_{x\to 0} \frac{\frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{4} - \cdots}{x^2}$$

$$= \frac{1}{2} + \lim_{x\to 0} \left(-\frac{x}{3} + \frac{x^2}{4} - \cdots\right) = \frac{1}{2}$$

2. $\lim_{x\to 0} \frac{\sin x - x + \frac{1}{6}x^3}{x^5}$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\lim_{x\to 0} \frac{\sin x - x + \frac{1}{6}x^3}{x^5} = \lim_{x\to 0} \frac{\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots\right) - x + \frac{1}{6}x^3}{x^5} = \lim_{x\to 0} \frac{\left(\frac{x^5}{5!} - \frac{x^7}{7!} + \cdots\right)}{x^5}$$

$$= \lim_{x\to 0} \left(\frac{1}{5!} - \frac{x^2}{7!} + \cdots\right) = \frac{1}{5!} = \frac{1}{1\times 2\times 3\times 4\times 5} = \frac{1}{120}$$

3. $\lim_{x\to 0} \frac{\tan x - x}{x^3}$

$$\tan x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$$

$$\lim_{x\to 0} \frac{\tan x - x}{x^3} = \lim_{x\to 0} \frac{\left(x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots\right) - x}{x^3}$$

$$= \lim_{x\to 0} \frac{\left(\frac{x^3}{3!} + \frac{x^5}{5!} + \cdots\right)}{x^3}$$

$$= \lim_{x\to 0} \left(\frac{1}{3!} + \frac{x^2}{5!} + \cdots\right)$$

$$= \frac{1}{3!}$$

$$= \frac{1}{6}$$