Math 1B Absolute convergence; ratio & root tests Mar 6 2020

1. Determine whether the series converges absolutely or conditionally.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$

(b) $\sum_{n=1}^{\infty} \frac{\sin(n)}{2^n}$

(c) $\sum_{n=1}^{\infty} \frac{1}{2^n}$

2. Convergent or divergent? (If convergent, is it absolutely so?)

(a) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{3^n}{2^n n^3}$

(b) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{2^n}{3^n n^3}$

(c) $\sum_{n=1}^{\infty} \frac{1}{n!}$

(d) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$

(e) $\sum_{n=1}^{\infty} \left(\frac{n^2+1}{2n^2+1}\right)^n$

(f) $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

Theorem If $\sum_{n=1}^{\infty} |a_n|$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

Def

$\sum_{n=1}^{\infty} \lvert a_n \rvert$ converges? \ $\sum_{n=1}^{\infty} a_n$ converges?	Y	N
Y	$\sum_{n=1}^{\infty} a_n$ is absolutely convergent	
N	$\sum_{n=1}^{\infty} a_n$ is conditionally convergent	$\sum_{n=1}^{\infty} a_n$ is divergent

The ratio test (1) If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$, then $\sum_{n=1}^{\infty} a_n$ is absolutely conv.

(2) If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| > 1$ or $= \infty$, then $\sum_{n=1}^{\infty} a_n$ is divergent.

(else inconclusive)

The root test (1) If $\lim_{n\to\infty} |a_n|^{1/n} < 1$, then $\sum_{n=1}^{\infty} a_n$ is absolutely conv.

(2) If $\lim_{n\to\infty} |a_n|^{1/n} > 1$, then $\sum_{n=1}^{\infty} a_n$ is divergent.

(else inconclusive)

[**Rearrangements** If $\sum_{n=1}^{\infty} a_n$ is conditionally conv. and $r \in \mathbb{R}$, then there is a rearrangement $\{b_n\}$ of $\{a_n\}$ with $\sum_{n=1}^{\infty} b_n = r$. (!)]