Math 1B
Disc #2

Integration of rational functions

Jan 29
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Review

1. Every polynomial with real coefficients can be factored into linear ($ax+b$, $a \neq 0$) and irreducible quadratic (ax^2+bx+c, $a \neq 0$, $b^2-4ac<0$) factors. Why can every cubic poly. be factored? (Recall the Intermediate Value Theorem from 1A.)
2. Partial fraction decomposition of $\frac{f(x)}{g(x)}$:
   (a) If needed, do long division to get $\frac{f(x)}{g(x)} = q(x) + \underbrace{\frac{r(x)}{g(x)}}$, $\deg r < \deg g$
   continue with this rational function
   (b) Factor $g(x)$
   (c) write down "ansatz" for $\frac{f(x)}{g(x)}$:

   ex $g(x) = (x-1)(x+2)^3(x^2+x+1)(x^2+1)^2$

$$\frac{f(x)}{g(x)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2} + \frac{D}{(x+2)^3} + \frac{Ex+F}{x^2+x+1} + \frac{Gx+H}{x^2+1} + \frac{Ix+J}{(x^2+1)^2}$$

   (d) Solve for $A, B, C, \ldots$ (A hint: multiply both sides by $g(x)$, then plug in roots of $g(x)$ one at a time.)
   (e) Integrate each factor by an appropriate method.
   For quadratic terms, complete the square and u-substitute.

Building blocks

1. Complete the square: (a) x^2+5x+3
   (b) $2x^2-4x+12$

2. Find partial fraction decompositions:

   (a) $\dfrac{x^2+2x-1}{2x^3+3x^2-2x}$

   (b) $\dfrac{4x}{x^3-x^2-x+1}$

(c) $\dfrac{2x^2 - x + 4}{x^3 + 4x}$

(d) $\dfrac{x^3 + x^2 + 1}{x(x-1)(x^2+x+1)(x^2+1)^3}$

Integrals

(a) $\displaystyle\int \frac{3x^2 + 6x + 2}{x^2 + 3x + 2}\, dx$

(b) $\displaystyle\int \frac{x + 4}{x^2 + 2x + 5}\, dx$

(c) $\displaystyle\int \frac{1}{\sqrt{x} + \sqrt[3]{x}}\, dx$