$y'' + y = 0$

$\lambda^2 + 1 = 0$

$\lambda = \pm i$

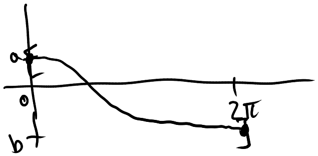
$\lambda = \alpha \pm \beta i$

$y = e^{\alpha t}(c_1 \cos \beta t + c_2 \sin \beta t)$

$\alpha = 0, \quad \beta = 1$

$y = c_1 \cos(t) + c_2 \sin(t)$

(A) $y(0) = a \quad y(2\pi) = b$



$$\begin{cases} a = y(0) = c_1 + 0 \\ b = y(2\pi) = c_1 + 0 \end{cases}$$

$a = c_1 = b$

underconstrained
(c_2 is free)

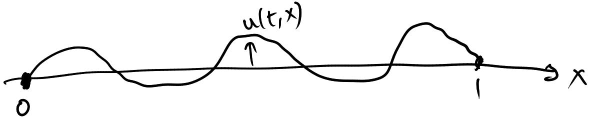
So: soln if $a = b$,

$y = a\cos(t) + c_2 \sin(t)$, c_2 any constant

(B) $y(0) = a \quad y(\pi/2) = b$

$$\begin{cases} a = y(0) = c_1 \\ b = y(\pi/2) = 0 + c_2 \end{cases}$$

so: soln is $y = a\cos t + b \sin t$



$u(t,x)$

$$\begin{cases} u(t,0) = 0 \\ u(t,1) = 0 \end{cases}$$

$u(0,x) = f(x)$

Fourier analysis

Non-homogeneous linear diff. eqns with constant coeffs

* Method of undetermined coeffs

$y' - 2y = 0$

$\lambda - 2 = 0$

$\lambda = 2$

$y = ce^{2t}$

$y' - 2y = G(t)$

$G(t)$ is made of e^{ct}, $\cos(ct)$, $\sin(ct)$, t

$G(t) = e^t \cos(2t)$

or $= (1 + 3t + t^2)e^{3t}$

or $= 2t$

or $= \sin(2t) + 3\cos(4t)$

ex $\underbrace{y' - 2y}_{p=-2} = \underbrace{e^{3t}}_{Q}$

$I = e^{\int p\,dt} = e^{-2t}$

$y = \frac{1}{I}\int IQ\,dt$

$= e^{2t}\int e^t\,dt = e^{2t}(e^t + c)$

$y = e^{3t} + ce^{2t}$

ex $y' - 2y = e^{4t}$

$y_p = Ae^{4t}$

$y_p' - 2y_p = 4Ae^{4t} - 2Ae^{4t} = 2Ae^{4t}$

$2A = 1$ so $A = \frac{1}{2}$

$\boxed{y = \frac{1}{2}Ae^{4t} + ce^{2t}}$

ex $y' - 2y = e^{2t}$

$y_p = Ae^{2t}$!

$y_p' - 2y_p = 2Ae^{2t} - 2\cdot Ae^{2t} = 0$

A fix: $\lambda - 2 = 0$

$\lambda = 2$

e^{2t} assoc to $\lambda = 2$

$y_p = Ate^{2t} \cancel{+ Be^{2t}}$

$y_p' - 2y_p = A(t\cdot 2e^{2t} + e^{2t}) - 2Ate^{2t}$

$= Ae^{2t}$ want to $= e^{2t}$ (RHS)

so $A = 1$ $\quad y_p = te^{2t}$

$y = te^{2t} + Ce^{2t}$

$P = -2 \quad Q = e^{2t}$

$I = e^{-2t}$

$y = \frac{1}{I}\int IQ\,dt = e^{2t}\int 1\,dt$

$= e^{2t}(t + C)$

$= te^{2t} + Ce^{2t}$

ex $\underbrace{y' - 2y}_{\lambda = 2} = \underbrace{te^{2t}}_{\lambda = 2,2}$

$y_p = At^2e^{2t} + \cancel{Bte^{2t}} + \cancel{Ce^{2t}}$

$y_p' = A(2t^2e^{2t} + 2te^{2t})$

$y_p' - 2y_p = 2Ate^{2t} = te^{2t}$

$A = \frac{1}{2}$

$y = \frac{1}{2}t^2e^{2t} + Ce^{2t}$

Annihilator method

ex

$$y'' - 3y' + 2y = \underbrace{e^{4t}}_{\text{assoc to } \lambda = 4}$$

$$\frac{d}{dt} e^{4t} = 4e^{4t}$$

$$y' = 4y$$

$$y' - 4y = 0$$

$$\lambda - 4 = 0$$

$$\lambda = 4$$

$$\lambda^2 - 3\lambda + 2 = 0$$

$$(\lambda - 2)(\lambda - 1) = 0$$

$$\lambda = 1, 2$$

$$y_h = c_1 e^t + c_2 e^{2t}$$

$$y_p = A e^{4t} \longleftarrow$$

$$y_p' = 4Ae^{4t}$$

$$y_p'' = 16Ae^{4t}$$

$$y_p'' - 3y_p' + 2y_p = e^{4t}(16A - 3 \cdot 4A + 2 \cdot A)$$

$$= e^{4t} \cdot 6A = e^{4t}$$

so $A = \frac{1}{6}$

$$\boxed{y = \frac{1}{6} e^{4t} + c_1 e^t + c_2 e^{2t}}$$

ex

$$\underbrace{y'' - 3y' + 2y}_{\lambda = 1, 2} = \underbrace{\cos(t)}_{\lambda = \pm i}$$

$$y_p = A\cos(t) + B\sin(t)$$

$$y_p' =$$

$$y_p'' = \ldots$$

$$y_p'' - y_p' + 2y_p = \ldots$$