$$f_0 \quad f_1 \quad f_2 \quad f_3 \quad \cdots$$

$$\{f_n\} = 1, 1, 2, 3, 5, 8, 13, \ldots$$

Formal power series $\displaystyle F(x) = \sum_{n=0}^{\infty} f_n x^n = f_0 + f_1 x + f_2 x^2 + \cdots$

"holds onto" the series

$$\begin{array}{rcccccc}
F(x) = & 1 & + x & + 2x^2 & + 3x^3 & + 5x^4 & + 8x^5 \\
x\,F(x) = & & x & + x^2 & + 2x^3 & + 3x^4 & + 5x^5 \\
-\ x^2 F(x) = & & & x^2 & + x^3 & + 2x^4 & + 3x^5 \\
\hline
(1-x-x^2)F(x) = & 1 & + 0 & + 0 & + 0 & + 0 & + 0
\end{array}$$

$$F(x) = \frac{1}{1-x-x^2} = \frac{1}{\sqrt5}\left(\frac{1}{\frac{-1+\sqrt5}{2} - x} - \frac{1}{\frac{-1-\sqrt5}{2} - x}\right)$$

$\varphi = \frac{1+\sqrt5}{2} = 1.618\ldots$

$\varphi^{-1} = \frac{-1+\sqrt5}{2} = 0.618\ldots$

$$= \frac{1}{\sqrt5}\left(\frac{1}{\varphi^{-1} - x} + \frac{1}{+\varphi + x}\right)$$

$$= \frac{1}{\sqrt5}\left(\frac{\varphi}{1-\varphi x} + \frac{\varphi^{-1}}{1+\varphi^{-1}x}\right)$$

$$\frac{\varphi^{-1}}{1-(-\varphi^{-1}x)} = \frac{a}{1-r} = \sum ar^n$$

$$\frac{\varphi}{1-\varphi x} \qquad \frac{\varphi^{-1}}{1+\varphi^{-1}x}$$

$$-1 < \varphi x < 1 \qquad -1 < -\varphi^{-1}x < 1$$

$$-\varphi^{-1} < x < \varphi^{-1} \qquad -\varphi < x < \varphi$$

$$\boxed{-\varphi^{-1} < x < \varphi^{-1}}$$

$$= \frac{1}{\sqrt5}\left(\sum_{n=0}^{\infty} \varphi\cdot\varphi^n x^n + \underbrace{\sum_{n=0}^{\infty} \varphi^{-1}(-\varphi^{-1}x)^n}_{\sum_{n=0}^{\infty} \varphi^{-1}(-1)^n(\varphi^{-1})^n x^n}\right)$$

$$= \sum_{n=0}^{\infty} \frac{\varphi^{n+1} + (-1)^n \varphi^{-n-1}}{\sqrt5} x^n$$

$$\boxed{f_n = \frac{\varphi^{n+1} + (-1)^n \varphi^{-n-1}}{\sqrt5}}$$

$$\varphi^{-n-1} = (\varphi^{-1})^{n+1} \approx (0.618)^{n+1} \xrightarrow[n\to\infty]{} 0$$

$$f_n \approx \frac{\varphi^{n+1}}{\sqrt5}$$

0	0.723607	$\to 1$
1	1.17082	$\to 1$
2	1.89443	$\to 2$
3	3.06525	$\to 3$
4	4.95967	$\to 5$
5	8.02492	$\to 8$
6	12.9846	$\to 13$
7	21.0095	$\to 21$
8	33.9941	
9	55.0036	
10	88.9978	

$$f_n = \text{round}\left(\frac{\varphi^{n+1}}{\sqrt5}\right)$$

$$F\left(\frac{1}{100}\right) = \frac{1}{1-\frac{1}{100}-\frac{1}{100^2}} = \frac{10000}{9899}$$

$$= 1 + \frac{1}{100} + \frac{2}{100^2} + \frac{3}{100^3} + \frac{5}{100^4} + \cdots$$

$$= 1.0102030508 13\ldots$$

$$F\left(\frac{1}{1000}\right) = 1.001002003005\ldots$$

Taylor series of e^{-2x} centered at $a=3$

n	$f^{(n)}(x)$	$f^{(n)}(3)$
0	e^{-2x}	e^{-6}
1	$-2e^{-2x}$	$-2e^{-6}$
2	$4e^{-2x}$	$4e^{-6}$
3	$-8e^{-2x}$	$-8e^{-6}$
4	$16e^{-2x}$	$16e^{-6}$
$\vdots$	$\vdots$	$\vdots$
n	$(-2)^n e^{-2x}$	$(-2)^n e^{-6}$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(3)}{n!}(x-3)^n$$
$$= \sum_{n=0}^{\infty} \frac{(-2)^n e^{6}}{n!}(x-3)^n$$

$a=0$ center: (Maclaurin series)

$$f(x) = \sum_{n=0}^{\infty} \frac{(-2)^n}{n!}(x-0)^n$$
$$= \sum_{n=0}^{\infty} \frac{(-2)^n}{n!} x^n$$

ex

$$e^{x-3} = \sum_{n=0}^{\infty} \frac{(x-3)^n}{n!}$$
$$e^{x} = e^{-3}e^{x-3} = \sum_{n=0}^{\infty} \frac{e^{-3}(x-3)^n}{n!}$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$
$$e^{-2x} = \sum_{n=0}^{\infty} \frac{(-2x)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-2)^n x^n}{n!}$$

$$e^{-2x} = e^{-2(x-3)-6} = e^{-6}e^{-2(x-3)} = e^{-6}\sum_{n=0}^{\infty} \frac{(-2(x-3))^n}{n!} = e^{-6}\sum_{n=0}^{\infty} \frac{(-2)^n (x-3)^n}{n!}$$
$$= \sum_{n=0}^{\infty} \frac{(-2)^n e^{-6}}{n!}(x-3)^n$$

$$x^3 e^{-x^2} = x^3 \sum_{n=0}^{\infty} \frac{(-x^2)^n}{n!} = x^3 \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+3}}{n!}$$
$$= \frac{x^3}{0!} - \frac{x^5}{1!} + \frac{x^7}{2!} - \frac{x^9}{3!} + \frac{x^{11}}{4!}$$

$$e^x = \frac{1}{0!} + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^n}{n!} + \cdots$$

$$e^{-x^2} = \frac{1}{0!} + \frac{-x^2}{1!} + \frac{x^4}{2!} + \frac{-x^6}{3!} + \frac{x^8}{4!} + \cdots + \frac{(-x^2)^n}{n!} + \cdots$$

$$x^3 e^{-x^2} = \frac{x^3}{0!} - \frac{x^5}{1!} + \frac{x^7}{2!} - \frac{x^9}{3!} + \frac{x^{11}}{4!} + \cdots + \frac{x^3(-x^2)^n}{n!} + \cdots$$