Abstract Linear Algebra - Problem Set 1 Instructor: Katalin Berlow

The Homework will be out of 10 points total.

- 1. (3 points) Let S be a set of natural numbers. Consider the statement "If any number in S is odd, then 2 is not in S."
 - (a) Write the contrapositive of this statement.
 - (b) Write the converse of this statement.
 - (c) Write the negation of this statement.
- 2. (1 points) List all subsets of the set $\{0, 1, 2\}$.
- 3. (1 point) Consider the following graph.



Write the adjacency matrix for this graph.

- 4. (1 point) How would the adjacency matrix of a graph change if we add self-loops at each vertex?
- 5. (4 points) Let V and W be vector spaces over the field \mathbb{F} . Prove that the product $V \times W$ is also a vector space with the same field \mathbb{F} .

(In the product space, we have $(v_1, w_1) + (v_2, w_2) = (v_1 + v_2, w_1 + w_2)$ and $c \cdot (v, w) = (cv, cw)$).

Extra Credit

6. (3 bonus points) Let G be any graph. Let A be its adjacency matrix. What do the entries of A^n represent? Prove your answer.