Math 53 DIS 103/105 Worksheet: 10.2/10.3

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Review:

(a) learning how to find an equation of a tangent line.

(b) Examples we've done: §10.2 (7), (13), (27), *(30) using

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}; \qquad \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

(c) For polar curves $r = f(\theta)$,

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{d(r\sin\theta)}{d\theta}}{\frac{d(r\cos\theta)}{d\theta}} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

Examples we have done: §10.3 (20), *(57)

Today:

§10.1 (31) Show that the parametric equation $x = x_1 + (x_2 - x_1)t, y = y_1 + (y_2 - y_1)t, 0 \le t \le 1$ describes the line segment between (x_1, y_1) and (x_2, y_2).

§10.1 *(34) Find the parametric equations for $9x^2 - 18x + 4y^2 - 16y - 11 = 0$.

§10.3 (11) Sketch the region satisfying $2 < r < 3, \frac{5\pi}{3} \le \theta \le \frac{7\pi}{3}$.

§10.3 (14) Find out the distance between the points with polar coordinates (r_1, θ_1) and (r_2, θ_2).

§10.3 *(22) Find a polar equation for the curve $xy = 0$.

§10.3 *(44) Sketch the curve $r\theta = 1, \frac{\pi}{2} \le \theta \le 4\pi$. Remember that last time we have solved that the equation of the tangent line at $\theta = \pi$ is $y = -\pi x - 1$. Try to sketch it and make it look like a tangent line. What will happen if we consider $-4\pi \le \theta < 0$?

Extra:

§10.2 (73) A string is wound around a circle and then unwound while being held taut. The curve traced by the point P at the end of the string is called the **involute** of the circle. If the circle has radius r and center $(0, 0)$ and the initial position of P is $(r, 0)$. Find the parametric equations of the involute.