1. (a) $P = \frac{F}{A} = \frac{mg}{A} = \frac{\rho A d g}{A} = \rho g d = 1000kg/m^3 \cdot 9.8m/s^2 \cdot 3ft \cdot 0.3m/ft = 8820kg/ms^2$.
   (b) $F = PA = 882kg/ms^2 \cdot 5ft \cdot 2ft \cdot (0.3m/ft)^2 = 7938kg \cdot m/s^2$.
   (c) $F = \int_0^3 \rho g h w dh = \frac{\rho g h^2 \cdot w}{2}\Big|_0^3 = \frac{1000kg/m^3 \cdot 9.8m/s^2 \cdot (3ft)^2 \cdot 2ft \cdot (0.3m/ft)^3}{2} = 2381.4kg \cdot m/s^2$.

7. $\sum_{i=1}^n \rho g x_i^* \cdot \frac{2(\sqrt{3}-x_i^*)}{\sqrt{3}}\Delta x$. $\int_0^{\sqrt{3}} \rho g h \frac{2(\sqrt{3}-h)}{\sqrt{3}} dh = \rho g \left(h^2 - \frac{2h^3}{3\sqrt{3}}\right)_0^{\sqrt{3}} = 1000kg/m^3 \cdot 9.8m/s^2 \cdot (3-2)m^3 = 9800kg \cdot m/s^2 = 9800N$.

12. (a) $\int_0^6 \rho g h \cdot 2\sqrt{3^2-(3-h)^2}dh = 2\rho g \int_{-3}^3 (u+3)\sqrt{9-u^2}du = 2\rho g \int_{-\pi/2}^{\pi/2}(3\sin\theta + 3)3\cos\theta d(3\sin\theta) = 54\rho g \int_{-\pi/2}^{\pi/2}(\sin\theta + 1)\cos^2\theta d\theta$, where $\int_{-\pi/2}^{\pi/2}\sin\theta\cos^2\theta d\theta = \frac{-\cos^3\theta}{3}\Big|_{-\pi/2}^{\pi/2} = 0$ and $\int_{-\pi/2}^{\pi/2}\cos^2\theta d\theta = \frac{\theta+\sin\theta\cos\theta}{2}\Big|_{-\pi/2}^{\pi/2} = \frac{\pi}{2}$. Hence

$$\int_0^6 \rho g h \cdot 2\sqrt{3^2-(3-h)^2}dh = 27\pi \cdot 64.6 \cdot 9.8 \cdot 0.3lb \cdot ft/s^2 \approx 16110lb \cdot ft/s^2$$

   (b) $\int_0^3 \rho g h 2\sqrt{9-h^2}dh = 2\rho g \int_0^{\pi/2}(3\sin\theta)3\cos\theta d(3\sin\theta) = 54\rho g \int_0^{\pi/2}\sin\theta\cos^2\theta d\theta = 54\rho g \frac{-\cos^3\theta}{3}\Big|_0^{\pi/2} = 18\rho g = 18 \cdot 64.6 \cdot 9.8 \cdot 0.3lb \cdot ft/s^2 \approx 3419lb \cdot ft/s^2$.

23. $M_x = 4(-3) + 2(1) + 4(5) = 10$. $M_y = 4(2) + 2(-3) + 4(3) = 14$. $m = 4 + 2 + 4 = 10$. $\bar{y} = \frac{M_x}{m} = 1$ and $\bar{x} = \frac{M_y}{m} = 1.4$.

27. $\bar{x} = \frac{\int_0^1 xe^x dx}{\int_0^1 e^x dx} = \frac{xe^x|_0^1 - \int_0^1 e^x dx}{\int_0^1 e^x dx} = \frac{e}{e^x|_0^1} - 1 = \frac{1}{e-1}$. $\bar{y} = \frac{\int_0^1 \frac{1}{2}(e^x)^2 dx}{\int_0^1 e^x dx} = \frac{\frac{e^{2x}}{4}\Big|_0^1}{e-1} = \frac{e+1}{4}$.

35.

$$M_x = \int_0^4 \frac{1}{2}\frac{3x}{4} \cdot \rho\frac{3x}{4}dx = \frac{90}{32}\int_0^4 x^2 dx = \frac{45}{16}\frac{4^3}{3} = 60$$

$$M_y = \int_0^4 x \cdot \rho\frac{3x}{4}dx = \frac{30}{4}\int_0^4 x^2 dx = \frac{30}{4}\frac{4^3}{3} = 160$$

$$\bar{x} = \frac{M_y}{m} = \frac{160}{10 \cdot \frac{1}{2} \cdot 4 \cdot 3} = \frac{160}{60} = \frac{8}{3}$$

$$\bar{y} = \frac{M_x}{m} = \frac{60}{60} = 1$$

39. We can always assume $a < 0 < c$. $y = f(x) = \frac{-b}{a}x + b$, $y = g(x) = \frac{-b}{c}x + b$.

$$\begin{aligned} M_y &= \int_a^0 xf(x)dx + \int_0^c xg(x)dx \\ &= \frac{-bx^3}{3a} + \frac{bx^2}{2}\Big|_a^0 + \frac{-bx^3}{3c} + \frac{bx^2}{2}\Big|_0^c = \frac{b(c^2-a^2)}{6} \end{aligned}$$

$$
\begin{aligned}
M_x &= \int_a^0 \frac{f(x)^2}{2} dx + \int_0^c \frac{g(x)^2}{2} dx \\
&= \frac{1}{2} \int_0^b y^2 d\left(\frac{-a}{b}(y-b)\right) + \frac{1}{2} \int_b^0 y^2 d\left(\frac{-c}{b}(y-b)\right) \\
&= \frac{1}{2}\left[\frac{-a}{b}\frac{y^3}{3}\bigg|_0^b + \frac{-c}{b}\frac{y^3}{3}\bigg|_b^0\right] = \frac{b^2(c-a)}{6}
\end{aligned}
$$

Hence $\bar{x} = \frac{M_y}{\frac{1}{2}b(c-a)} = \frac{c+a}{3}$. $\bar{y} = \frac{M_x}{\frac{1}{2}b(c-a)} = \frac{b}{3}$. And the intersection of medians is just $\left(\frac{a+0+c}{3}, \frac{0+b+0}{3}\right)$. Hence they are the same.

44. $\frac{1}{2}\pi r^2 \cdot 2\pi \left(\frac{4r}{3\pi}\right) = \frac{4}{3}\pi r^3$.

45. $\frac{1}{2}hr \cdot 2\pi \frac{r}{3} = \frac{\pi}{3} r^2 h$.