3. $\int_0^{\pi/2} \sin^7\theta\cos^5\theta d\theta = \int_0^{\pi/2} \sin^7\theta(1-\sin^2\theta)^2 d\sin\theta = \int_0^1 u^7(1-u^2)^2 du = \left[\frac{u^8}{8} - 2\frac{u^{10}}{10} + \frac{u^{12}}{12}\right]_0^1 = \frac{1}{120}$.

7. $\int_0^{\pi/2} \cos^2\theta d\theta = \int_0^{\pi/2} \frac{1}{2}\left(\cos(2\theta)+1\right) d\theta = \left[\frac{1}{4}\sin(2\theta) + \frac{\theta}{2}\right]_0^{\pi/2} = \frac{\pi}{4}$.

11. $\int_0^{\pi/2} \sin^2 x\cos^2 x dx = \int_0^{\pi/2} \frac{1}{4}\sin^2(2x) dx = \int_0^{\pi/2} \frac{1}{8}\left(1-\cos(4x)\right) dx = \left[\frac{x}{8} - \frac{\sin(4x)}{32}\right]_0^{\pi/2} = \frac{\pi}{16}$.

13. $\int t\sin^2 t dt = \int \frac{t - t\cos(2t)}{2} dt = \frac{t^2}{4} - \left[\frac{t\sin(2t)}{4} - \int \frac{\sin(2t)}{4} dt\right] = \frac{t^2}{4} - \frac{t\sin(2t)}{4} - \frac{\cos(2t)}{8} + C$.

23. $\int \tan^2 x dx = \int (\sec^2 x - 1) dx = \tan x - x + C$.

27. $\int_0^{\pi/3} \tan^5 x\sec^4 x dx = \int_0^{\pi/3} \tan^5 x(1+\tan^2 x) d\tan x = \int_0^{\sqrt{3}} u^5(1+u^2) du = \left[\frac{u^6}{6} + \frac{u^8}{8}\right]_0^{\sqrt{3}} = \frac{117}{8}$.

41. $\int \sin(8x)\cos(5x) dx = \frac{1}{2}\int \sin(13x) + \sin(3x) dx = \frac{-\cos(13x)}{26} + \frac{-\cos(3x)}{6} + C$.

56. (a) $\int u(-du) = \frac{-u^2}{2} = \frac{-\cos^2 x}{2} + C$.

    (b) $\int u du = \frac{u^2}{2} = \frac{\sin^2 x}{2} + C'$. $C' = C - \frac{1}{2}$.

    (c) $\frac{1}{2}\int \sin(2x) dx = \frac{-\cos(2x)}{4} + C''$. $C'' = C - \frac{1}{4}$.

    (d) $\int \sin x\cos x dx = \sin x\sin x - \int \cos x\sin x dx \Rightarrow \int \sin x\cos x dx = \frac{\sin^2 x}{2} + C'''$. $C''' = C'$.

57. $\int_{-\pi/4}^{\pi/4} (\cos^2 x - \sin^2 x) dx = \int_{-\pi/4}^{\pi/4} \cos(2x) dx = \left.\frac{\sin(2x)}{2}\right|_{-\pi/4}^{\pi/4} = 1$.

61. $\int_{\pi/2}^{\pi} \pi(\sin x)^2 dx = \frac{\pi}{2}\int_{\pi/2}^{\pi} (1-\cos(2x)) dx = \frac{\pi}{2}\left[x - \frac{\sin(2x)}{2}\right]_{\pi/2}^{\pi} = \frac{\pi^2}{4}$.

68. If $m = n$, $\frac{1}{2}\int_{-\pi}^{\pi} (1-\cos(2mx)) dx = \frac{1}{2}\left[x - \frac{\sin(2mx)}{2m}\right]_{-\pi}^{\pi} = \pi$. If $m \neq n$, $\frac{1}{2}\int_{-\pi}^{\pi} (\cos(m-n)x - \cos(m+n)x) dx = \frac{1}{2}\left[\frac{\sin(m-n)x}{m-n} - \frac{\sin(m+n)x}{m+n}\right]_{-\pi}^{\pi} = 0$.

70. By 68., $\frac{1}{\pi}\int_{-\pi}^{\pi} \left[\sum_{n=1}^{N} a_n \sin(nx)\right] \sin(mx) dx = \frac{1}{\pi}\int_{-\pi}^{\pi} a_m \sin(mx)\sin(mx) dx = a_m$.