1. True-False Quiz: if it is true, explain why. If it is false, give an example that disproves the statement.

   (a) If neither $\lim_{x\to a} f(x)$ nor $\lim_{x\to a} g(x)$ exists, then $\lim_{x\to a}[f(x)+g(x)]$ does not exist.

   (b) $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$.

   (c) If $f(x) = (x^6 - x^4)^5$, then $f^{(31)}(x) = 0$.

   (d) If f and g are increasing on an interval I, then fg is increasing on I.

   (e) An equation of the tangent line to the parabola $y = x^2$ at $(-2, 4)$ is $y - 4 = 2x(x+2)$.

2. Consider the triangle with vertices $(2, 0)$, $(0, 2)$, $(-1, 1)$.

   (a) Show that the shortest distance from the point $(-1, 1)$ to an arbitrary straight line $Ax + By + C = 0$ is $\frac{|-A+B+C|}{\sqrt{A^2+B^2}}$. [Hint: this is an optimization problem.]

   (b) Use the result of (a) to find the area of the triangle.

   (c) Use calculus to find the area of the triangle. Is your result the same as what you get in (b)?

   (d) Calculate the volume of the solid obtained by rotating the triangle with respect to the x-axis.

3. We try to calculate $\int_0^1 2^x dx$ in different ways.

   (a) Find the antiderivative of 2^x. Then calculate $\int_0^1 2^x dx$.

   (b) Show that $\lim_{n\to\infty} \frac{1}{n(2^{1/n}-1)} = \frac{1}{\ln 2}$.

   (c) Express the integral as a limit of Riemann sums with left endpoints.

   (d) Use the result of (b) and (c) to show that the limit of Riemann sums coincides with your answer in (a).

4. Consider $y = \sin(x^2)$, where $0 \le x \le \sqrt{\pi}$.

   (a) Find the horizontal asymptotes, vertical asymptotes, absolute maximums, absolute minimums for this function.

   (b) Investigate the intervals of increase/decrease and the intervals of concavity. And also find the inflection points. [You may not be able to solve the inflection points directly. First show that if x is an inflection point, then $\cot(x^2) = 2x^2$. Suppose $z = x^2$. Use Newton's method with initial approximation $z_1 = \frac{\pi}{2}$ to find the third approximation z_3.]

   (c) Sketch the graph of this function using all datas above.

   (d) Compute the volume of the solid obtained by rotating the function about the y-axis.

5. Show that $\frac{\int_x^y f(t)dt}{y-x} = f(z)$ for some z between x and y. [Hint: Suppose $g(x) = \int_a^x f(t)dt$. Try to use mean value theorem for $g(x)$.]

6. Use $\epsilon - \delta$ argument to show that $\lim\limits_{x \to a} \int\limits_a^x t^2 dt = 0$.