# Discussion 10/3/18 Solutions

<u>Problem Set 1</u>

1) $$\frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right)$$

$$= \frac{(\cos x)(1)' - 1(\cos x)'}{\cos^2 x}$$

$$= \frac{(\cos x)(0) - (1)(-\sin x)}{\cos^2 x}$$

$$= \frac{\sin x}{\cos^2 x} = \sec x \tan x$$

$\left(\sec x = \frac{1}{\cos x},\ \tan x = \frac{\sin x}{\cos x}\right)$

$$\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$$

$$= \frac{(\cos x)(\sin x)' - (\sin x)(\cos x)'}{\cos^2 x}$$

$\cos^2 x + \sin^2 x = 1.$ $\longrightarrow$ $$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

2) $$f(x) = \frac{\sin x}{x^2}$$

$$f'(x) = \frac{x^2(\sin x)' - (\sin x)(x^2)'}{x^4}$$

$$= \frac{x^2\cos x - 2x\sin x}{x^4} = \frac{\cos x}{x^2} - \frac{2\sin x}{x^3}$$

Or use product rule.

$$f(x) = \sin x \cdot \frac{1}{x^2}$$

$$f'(x) = (\sin x)\left(-\frac{2}{x^3}\right) + \cos x \cdot \frac{1}{x^2} = \frac{\cos x}{x^2} - \frac{2\sin x}{x^3}$$

These agree!!

3) (a) $x^2 - 4 = 0$

$(x-2)(x+2) = 0$

$x = -2, 2$ $\qquad \boxed{x = -2, x = 2}$

(b) $\lim\limits_{x \to \infty} \dfrac{2x^2}{x^2-4} = \lim\limits_{x \to \infty} \dfrac{\frac{2x^2}{x^2}}{\frac{x^2-4}{x^2}}$

$= \lim\limits_{x \to \infty} \dfrac{2}{1 - \frac{4}{x^2}} = \dfrac{2}{1-0} = 2.$

$\boxed{y = 2}$

(c) $x^2 = 0 \Rightarrow \boxed{x = 0}$

(d) $f(x) = \dfrac{2x^2}{x^2-4}.$

$f'(x) = \dfrac{(x^2-4)(4x) - 2x^2(2x)}{(x^2-4)^2}$

$= \dfrac{4x^3 - 16x - 4x^3}{(x^2-4)^2}$

$= -\dfrac{16x}{(x^2-4)^2}$

VA: $x = -2, x = 2$, HA: $y = 0$,

zeros: $x = 0$.

5)

4) Cycles in 4s.

$\sin x - \cos x$

$\downarrow$

$\cos x + \sin x$ $1^{st}$ der.

$\downarrow$

$-\sin x + \cos x$ $2^{nd}$ der.

$\frac{1003}{4} = 250 \underline{R}$

$\downarrow$

$-\cos x - \sin x$ $3^{rd}$ der.

$\boxed{f^{(1003)}(x) = -\cos x - \sin x}$

$\downarrow$

$\sin x - \cos x$ $4^{th}$ der.

$f(x) = xe^x$
$f'(x) = xe^x + e^x$
$f''(x) = xe^x + 2e^x$
$f'''(x) = xe^x + 3e^x$

So $\boxed{f^{(1003)}(x) = xe^x + 1003e^x}$

5) $f(x) = \frac{\sin x}{x e^x}$

$$f'(x) = \frac{x e^x \cos x - \sin x \,(x e^x + e^x)}{(x e^x)^2}.$$

$g(x) = \sqrt{x}\, e^x \cos x$

$$g'(x) = \sqrt{x}\,(e^x \cos x)' + \frac{1}{2\sqrt{x}} e^x \cos x$$

$$= \sqrt{x}\, e^x \cos x - \sqrt{x}\, e^x \sin x + \frac{1}{2\sqrt{x}} e^x \cos x$$

$$h(x) = \frac{\sqrt[3]{x}}{(x^2+3x-1)e^x}$$

$$h'(x) = \frac{\frac{1}{3} x^{-2/3}(x^2+3x+1)e^x - \sqrt[3]{x}\left((x^2+3x-1)e^x + (2x+3)e^x\right)}{(x^2+3x-1)^2 e^{2x}}$$

<u>Problem Set 3</u>

6) $f(x) = \sqrt{x} \sin x$

$$f'(x) = \sqrt{x} \cos x + \frac{1}{2\sqrt{x}} \sin x$$

$$g(x) = \frac{1}{\sqrt{x}} \sin x$$

$$g'(x) = \frac{1}{\sqrt{x}} \cos x + \left(-\frac{1}{2} x^{-3/2}\right) \sin x$$

$\lim_{x \to \infty} f(x)$ DNE since oscillating with amplitude $\sqrt{x} \to \infty$ as $x \to \infty$.

Similarly, $\lim_{x \to \infty} f'(x)$ DNE because even though $\lim_{x \to \infty} \frac{1}{2\sqrt{x}} \sin x = 0$, $\lim_{x \to \infty} \sqrt{x} \cos x$ DNE.

$$\lim_{x\to\infty} g(x) = \lim_{x\to\infty} \frac{1}{\sqrt{x}} \sin x = 0$$

by Squeeze Theorem.

$$-1 \le \sin x \le 1$$

$$-\frac{1}{\sqrt{x}} \le \frac{1}{\sqrt{x}} \sin x \le \frac{1}{\sqrt{x}}$$

Since $\lim_{x\to\infty} \left(-\frac{1}{\sqrt{x}}\right) = \lim_{x\to\infty} \frac{1}{\sqrt{x}} = 0$,

$\lim_{x\to\infty} \frac{1}{\sqrt{x}} \sin x = 0$ by Squeeze Theorem.

Similarly, by applying the Squeeze Theorem to both terms in the sum in $g'$, we get that

$$\lim_{x\to\infty} g'(x) = 0.$$