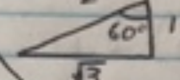
Discussion 10/1/18 Solutions

Problem Set 1

1) $f(x) = x - 2\sin x$
$f'(x) = 1 - 2\cos x$

$1 - 2\cos x = 0$
$\cos x = \frac{1}{2}$



$$\boxed{x = \frac{\pi}{3} + 2\pi k, \quad \frac{5\pi}{3} + 2\pi k \text{ for } k \text{ integer}}$$

Problem Set 2

2) $\lim_{b \to 2} \dfrac{b^{691} - 2^{691}}{b - 2} = f'(2)$ Let $f(x) = x^{691}$, $f'(x) = 691x^{6}$

$= \boxed{691 \cdot 2^{690}}$

3) (a) Let $f(x) = \sqrt[4]{x}$.

Find the equation of the tangent line to $x = 1$.

$f'(x) = \frac{1}{4} x^{-\frac{3}{4}} \Rightarrow f'(1) = \frac{1}{4}$.

$f(1) = 1$.

$y - 1 = \frac{1}{4}(x - 1) \Rightarrow y = \frac{1}{4}x + \frac{3}{4}$

Plug in $x = 1.003$.

$y = \frac{1}{4}(1.003) + \frac{3}{4} = \boxed{1.001}$

(b) Let $f(x) = e^x$.

Find the equation of the tangent line to f at $x=0$.

$f'(x) = e^x$ $\quad$ $f'(0) = 1$.
$f(0) = 1$.

$y = x+1$. $\quad$ $y = (0.05)+1$
$= \boxed{1.05}$

4) (a) $\overset{f=}{e^{2x}} \longrightarrow \overset{f'=}{2e^{2x}} \longrightarrow \overset{f''=}{4e^{2x}}$

So $f^{(10)}(x) = 2^{10}e^{2x}$.

(b) $f = x^{50} \longrightarrow f' = 50x^{49} \rightarrow f'' = 50 \cdot 49x^{48}$

So $f^{(50)}(x) = 50!\,x^0 = 50!$

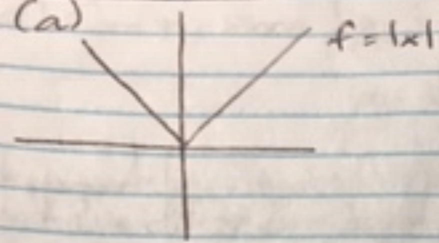
(c) $y = (x^{100}) + \underbrace{(e^x)}_{\text{derivative is always the same}}$

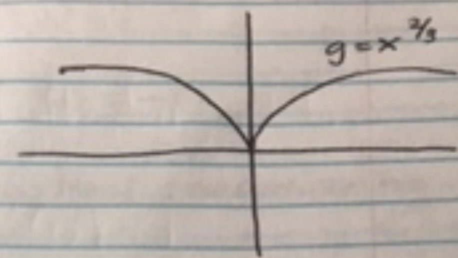
100^{th} derivative of x^{100} is $100!$ which is just a constant. So the 101^{th} derivative of x^{100} is 0.

So $f^{(101)}(x) = 0 + e^x = e^x$.

Problem Set 3

5) (a)





(b) $\lim_{h\to0^+} \frac{f(x+h)-f(x)}{h}\Big|_{x=0}$

$= \lim_{h\to0^+} \frac{f(h)-f(0)}{h} = \lim_{h\to0^+} \frac{h-0}{h} = 1.$

$\lim_{h\to0^-} \frac{f(h)-f(0)}{h} = \lim_{h\to0^-} \frac{-h-0}{h} = -1.$

So $\lim_{h\to0} \frac{f(h)-f(0)}{h}$ DNE, so f is not differentiable at 0.

$\lim_{h\to0} \frac{g(0+h)-g(0)}{h}$

$= \lim_{h\to0} \frac{h^{2/3}-0}{h} = \lim_{h\to0} \frac{1}{h^{1/3}}$ DNE

So g is not differentiable at $x=0$.

(c) $f'(x) = \begin{cases} 1 & \text{if } x>0 \\ -1 & \text{if } x<0 \end{cases}$

$g'(x) = \frac{2}{3}x^{-1/3}$ if $x\neq0$.

$\lim_{x\to0^+} f'(x) = 1$, $\lim_{x\to0^-} f'(x) = -1$

$\lim_{x\to0^+} g'(x) = \infty$, $\lim_{x\to0^-} g'(x) = -\infty$.

(d) Corner, since $\lim_{x \to 0^+} f'(x) = 1$

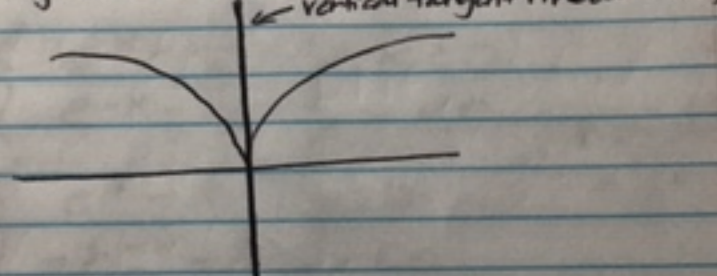
$$\lim_{x \to 0^-} f'(x) = -1$$

tangent lines in both directions are converging to have different slopes.

Vertical tangent, since $\lim_{x \to 0^+} g'(x) = \infty$

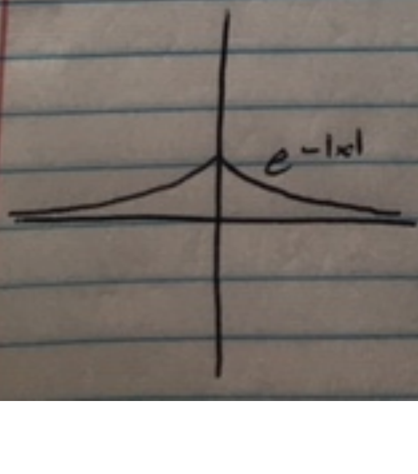
$$\lim_{x \to 0^-} g'(x) = -\infty$$

so tangent lines in both directions are approaching vertical lines. g actually has a vertical tangent at $x = 0$.



(e) $e^{-|x|} = \begin{cases} e^{-x} & \text{if } x \geq 0 \\ e^{x} & \text{if } x \leq 0 \end{cases}$ can check - not differentiable at $x = 0$.

$(e^{-|x|})' = \begin{cases} -e^{-x} & \text{if } x > 0 \\ e^{x} & \text{if } x < 0 \end{cases}$

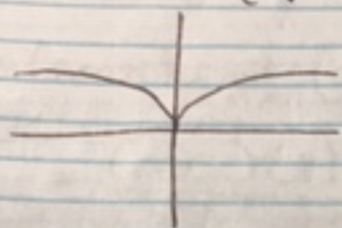


$$\lim_{x \to 0^+} (e^{-|x|})' = -1$$

$$\lim_{x \to 0^-} (e^{-|x|})' = 1$$

so this is a corner.

$$y = \sqrt{|x|} = \begin{cases} \sqrt{x} \text{ if } x \geq 0 \\ \sqrt{-x} \text{ if } x \leq 0 \end{cases}$$



can check –
not differentiable at $x = 0$.

$$y' = \begin{cases} \frac{1}{2\sqrt{x}} \text{ if } x > 0 \\ \frac{-1}{2\sqrt{-x}} \text{ if } x < 0 \end{cases}$$

$$\lim_{x \to 0^+} y' = \infty \quad \lim_{x \to 0^-} y' = -\infty.$$

So this function has a vertical tangent at $x = 0$.

$$y = e^{|x|} - \frac{1}{2}x^2$$

$$= \begin{cases} e^{x} - \frac{1}{2}x^2, & x \geq 0 \\ e^{-x} - \frac{1}{2}x^2, & x \leq 0. \end{cases}$$

not differentiable at $x = 0$.

$$y' = \begin{cases} e^{x} - x, & x > 0 \\ -e^{-x} - x, & x < 0 \end{cases}$$

$$\lim_{x \to 0^+} y' = 1, \quad \lim_{x \to 0^-} y' = -1$$

So this function has a corner at $x = 0$.