Math 1A: Discussion 9/21/2018 Review Session

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Question 1

Find the domain of the function

$$f(x) = \frac{\log(x^2 - 3x + 2)}{(\sin(x) + 1)(\sin(x) + 2)}$$

Question 2

Calculate the following limit and carefully justify using the limits laws and continuity.

$$\lim_{x \to 0} \sqrt{\frac{e^x + \arcsin(x)}{x^2 + 4}}$$

Question 3

Calculate the following limit.

$$\lim_{x \to 0} \arctan\left(x\cos\left(\frac{1}{\sqrt{x}}\right)\right)$$

Question 4

Calculate the following limit.

$$\lim_{h \to 0} \frac{\sqrt{\frac{1}{1+h^2}} - 1}{h}$$

Question 5

Evaluate the following quantities

$$\tan(\arccos(x))$$

$$e^{4\ln(\sqrt{3}) - \ln(6)}$$

Question 6

Consider the function

$$f(x) = 2x + 1 \text{ if } x \neq -1$$
$$f(-1) = 1$$

Show using an ϵ-δ proof that

$$\lim_{x \to 1} f(x) = -1$$

Consider the piecewise linear function $g(x)$ defined as

$$g(x) = \frac{1}{2}x - 1 \text{ if } x \geq 0$$
$$g(x) = -2x - 1 \text{ if } x < 0$$

Show using an ϵ-δ proof that

$$\lim_{x \to 0} g(x) = -1$$

Question 7

Describe how to get the graph of $y = 2e^{x-2} + 3$ from the graph of $y = \frac{1}{2}e^{x+2}$.

Question 8

Show that the function defined by

$$h(x) = x^2 \sin\left(\frac{1}{x}\right) \text{ if } x \neq 0$$

$$h(0) = 0$$

is continuous at $x = 0$. Is $h(x)$ also differentiable at $x = 0$? (Use the definition of the derivative)

Question 9

Calculate the following limits, and justify your answers.

$$\lim_{x \to e^-} \frac{x^2}{(1 - \ln(x))^5}$$

$$\lim_{x \to -\infty} \frac{e^x}{x^2 + 1}$$

$$\lim_{x \to \infty} \sqrt{x} - x$$