Midterm 2 Review Session

November 2, 2018

Question 1

Find

$$\frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right)^{\sqrt{x}}$$

$$\frac{d}{dx}(2+\cos(x))^{\sin(x)}$$

Question 2

Find the equation of the tangent and normal lines to

$$\arctan(\sqrt{xy}) - \frac{\pi}{4}x^2y = 0$$

at the point $(1, 1)$.

Question 3

Find the equation of all tangent lines to the hyperbola

$$x^2 - y^2 = 1$$

that pass through the origin.

Question 4

Evaluate

$$\lim_{x\to\infty}(e^x - x^2 - \ln(x))$$

Question 5 (Midterm 2, Fall 2000)

Evaluate

$$\lim_{x\to 1}\frac{e^x - e}{x^2 - 1}$$

$$\lim_{x\to\infty}\frac{\tan(x) - x + \pi}{x^3}$$

$$\lim_{x\to\infty}(e^{2x}+1)^{1/x}$$

Question 6

Compute

$$\frac{d}{dx}\left(3\sec^2(\arcsin(x^2))\right)$$

$$\frac{d}{dx}(\log_2(\log_3(\csc(x)))$$

$$\frac{d}{dx}\text{arccot}\left(\sqrt{1-\sqrt[3]{x}}\right)$$

Question 7

Sketch the graph of

$$f(x) = \frac{1}{7}x^{7/3} - x^{1/3}$$

$$f(x) = \frac{4x^3 + 5x^2 + 2x}{x^2 + x}$$

(Midterm 2, Fall 2011)

$$f(x) = e^{-x^2+2x}$$

(Midterm 2, 2005)

$$f(x) = \frac{\sin(x)}{1+\cos(x)}$$

(Midterm 2, 2005)

$$f(x) = x^{1/x} \text{ for } x > 0$$

Question 8

Show that the equation

$$\sin^2(x) + 3x + 1 = -\arctan(x)$$

has exactly one solution.

Question 9

Show that the equation

$$\cos(x) - 2x^2 = c$$

has at most 2 real solutions, where c is any constant.

Question 10

(Midterm 2, Fall 2001) A rectangular playground is to be fenced off and divided in half by another fence parallel to one side of the playground. The total area of both halves is to be 600 square feet. Find the dimensions of the playground that will use the minimal amount of fencing.

Question 11

What is the shortest distance from the parabola

$$y = \frac{1}{4}x^2 + 1$$

to the point $(0, 2)$?

Question 12

Consider the curve $y = 1 - x^2$. What is the largest possible area of a rectangle R with vertices (x, y), $(-x, y)$, $(x, 0)$ and $(-x, 0)$, where y is nonnegative, and (x, y) is on the curve $y = 1 - x^2$? What are the dimensions of this rectangle of maximal area?

Question 13

I deposit D dollars in the bank. I can either invest the D dollars at a rate of 10 percent semiannually, or at a rate of 10 percent continuously. How long does it take for the amount that I have to double, for each option?

Question 14

At 2 PM, a baker takes out a hot loaf of banana bread from the oven and a stick of cold butter from the refrigerator. The ambient temperature is 25 degrees Celcius, the temperature of the hot banana bread is 125 degrees Celcius initially, and the temperature of the cold butter is 5 degrees initially. At 2:30 PM, the banana bread is now 84 degrees Celcius and the butter is now 20 degrees Celcius. Write a function $T(t)$ that gives how much hotter the banana bread is than the cold butter at a time t in hours after 2 PM.