Math 54 Quiz 7 Study Guide

October 22, 2019

Note: Please review the last two study guides and the last quiz, since the material covered there can be tested on the upcoming quiz.

Conceptual Questions

- Show that the dimension of the left nullspace of an m by n matrix A is m minus the dimension of the column space.
- True or False: There is a real-valued matrix A that is diagonalizable over the complex numbers, but is not diagonalizable over the real numbers.
- True or False: The set of diagonalizable 3 by 3 matrices is a subspace.
- True or False: Every matrix has at least one real eigenvalue.
- Show that the only vector that is orthogonal to itself is the zero vector.
- True or False: The characteristic polynomial of a matrix is unchanged under row reduction.
- True or False: Similar matrices have the same characteristic polynomial. (Recall two matrices A and B are similar if there is an invertible matrix M such that $A = MBM^{-1}$).
- Let W be a subspace of $\mathbb{R}^n$. What is $(W^{\perp})^{\perp}$?

Problems

Problem 1

Suppose that for a matrix A,

$$p(A) = 0$$

for $p(x) = (1-x)^2(2-x)$. Find two possibilities for such a matrix A, one that is diagonalizable, and one that is not.

Problem 2

The trace of a 3 by 3 matrix A is -1, and its determinant is -8. One of its eigenvalues is 1. Is A diagonalizable over the real numbers? Is A diagonalizable over the complex numbers? Find all λ such that $\lim_{n\to\infty} \lambda^n A^n$ exists.

Problem 3

Find all possible real numbers a, b, and c such that the matrix

$$A = \begin{bmatrix} 1 & a & c \\ 0 & 2 & 0 \\ 0 & b & 1 \end{bmatrix}$$

is diagonalizable over the real numbers.

Problem 4

Let $W = \text{span}\{(1,2,1,0), (-1,0,1,1)\}$. Find a basis for $W^\perp$.

Problem 5

Let a, b, c be real numbers that are not all zero. Let W be the subspace given by the plane $ax_1 + bx_2 + cx_3 = 0$ in $\mathbb{R}^3$. Find a basis for $W^\perp$.

(Hint: This problem can be done relatively quickly without calculations, by using the fact that $ax_1 + bx_2 + cx_3 = \langle (a,b,c), (x_1,x_2,x_3) \rangle$. But you must justify why your resulting answer is a basis, perhaps by using some argument involving computation of dimensions.)