Math 54 Quiz 6 Study Guide (Continued)

October 15, 2019

Conceptual Questions

- Show that every matrix of odd dimension has at least one real eigenvalue. (Hint: complex conjugates.) Show the same is not true for matrices of even dimension. (Hint: Consider the matrix $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$)
- Is every upper triangular matrix diagonalizable?
- True or False: Every matrix with repeated eigenvalues is not diagonalizable.
- Let A be a matrix whose entries in each row sum to the same number. (For example, $A = \begin{bmatrix} 1 & 2 & 1 \\ 4 & 0 & 0 \\ -1 & 6 & -1 \end{bmatrix}$ is such a matrix.) Find an eigenvalue and eigenvector of A.
- Show that every 3 by 3 matrix with a non-real eigenvalue is diagonalizable over the complex numbers.

Problems

Problem 1

Are the following matrices diagonalizable over the real numbers? Over the complex numbers? If so, write them as SDS^{-1} for a diagonal matrix D.

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & -3 & 0 \\ 3 & -5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}$$

Problem 2

Let A be a 3 by 3 matrix with trace 4. Suppose you know that one of the eigenvalues is $1-2i$. What is the determinant of A? What is the characteristic polynomial of A?

Problem 3

Consider the matrix

$$M = \begin{bmatrix} 1/2 & -1/2 & 0 \\ -1/2 & 0 & 1/2 \\ 0 & 1/2 & 1/2 \end{bmatrix}$$

Diagonalize M. Does $\lim_{n\to\infty} M^n$ exist? If so, compute the value of the limit.

Problem 4

Consider the matrix A from Problem 1. Find all real numbers λ such that $\lim_{n\to\infty} \lambda^n A^n$ exists.

Problem 5

Suppose that A has eigenvectors $(1,1,0)$, $(1,0,1)$, and $(0,1,-2)$ corresponding to eigenvalues -1, 0, and 1 respectively. Compute $A^{2019}(10,15,19)$.