Math 54 Quiz 6 Study Guide

October 8, 2019

Additional Reading and Practice: See Math N54 lecture notes on matrix multiplication and invertible matrices (June 28).

Conceptual Questions

- Show that 0 cannot be an eigenvalue of an invertible matrix.
- If λ is an eigenvalue of an invertible matrix A with eigenvector v (which is nonzero necessarily by the previous problem), find an eigenvalue and eigenvector of A^{-1}.
- If λ is an eigenvalue of the matrix A with eigenvector v, find an eigenvalue and eigenvector for A^k where k is any positive integer.
- Is every invertible matrix diagonalizable? (Hint: Look at Problem 1)
- Can a noninvertible matrix be diagonalizable?

Problems

Problem 1

Define the size k Jordan block with eigenvalue λ to be the matrix with λ along the main diagonal and ones on the diagonal immediately above the main diagonal. For example, the Jordan block with eigenvalue 3 of size 5 is

$$\begin{bmatrix} 3 & 1 & 0 & 0 & 0 \\ 0 & 3 & 1 & 0 & 0 \\ 0 & 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix}$$

Find the characteristic polynomial and all eigenvalues (and their geometric multiplicity and algebraic multiplicity) of a size k Jordan block with eigenvalue λ.

Problem 2

Let A be an invertible matrix and let M be a square matrix of the same size.

- Find a formula for $(AMA^{-1})^k$ where k is a positive integer.
- Show that $\det(AMA^{-1}) = \det(M)$
- Show that $\text{char}_{AMA^{-1}}(x) = \text{char}_M(x)$.

Problem 3

Find all eigenvalues and eigenvectors (with their geometric and algebraic multiplicities) of the following matrices.

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & 1 \\ 0 & 0 & 3 \end{bmatrix} \quad \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Problem 4

Find all k such that

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & k & 1 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

is diagoalizable. For every possible value of k, find the eigenvalues and eigenvectors of the matrix above and their geometric and algebraic multiplicities.