Math 54 Quiz 5

September 26, 2019

Part 1: True/False (15 points)

Directions: For each item, circle either True or False. (1 point each)

- (True/False) $\mathbf{v}_1$ and $\mathbf{v}_2$ are linearly independent if neither is a scalar multiple of the other.
- (True/False) $1, x, x^2, x^3$ is a basis for P_3.
- (True/False) The span of $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ in $M_{2\times 2}$ is all 2 by 2 matrices whose second row is all zero.

  $\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$ not in span
- (True/False) The vector $(2, 0, 1, 9)$ is in the left nullspace of the matrix $A = \begin{bmatrix} 1 & 4 & 0 \\ 9 & 12 & -3 \\ -2 & 1 & 0 \\ 0 & -1 & 0 \end{bmatrix}$.

  $\begin{bmatrix} 2 & 0 & 1 & 9 \end{bmatrix} \begin{bmatrix} 1 & 4 & 0 \\ 9 & 12 & -3 \\ -2 & 1 & 0 \\ 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$
- (True/False) The row space of $A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 2 & 2 & 0 & 0 \end{bmatrix}$ is a subspace of $\mathbb{R}^2$.

  $(\mathbb{R}^4)$
- (True/False) The column space of an invertible square n by n matrix must be $\mathbb{R}^n$.
- (True/False) The kernel of a linear transformation $T : \mathbb{R}^m \to \mathbb{R}^n$ is the set of all $\mathbf{x} \in \mathbb{R}^m$ such that $T(\mathbf{x}) = \mathbf{0}$.
- (True/False) There is a basis of P_9 that has 10 vectors in it.

  $\dim(P_9) = 10$
- (True/False) The set of noninvertible 2 by 2 matrices is a subspace of $M_{2\times 2}$. $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
- (True/False) The nullspace of an m by n matrix A is a subspace of $\mathbb{R}^n$.
- (True/False) The vectors $(-1, 0, 0, 0), (0, 0, 0, 2), (0, 3, 0, 0), (0, 0, -4, 0)$ form a basis for $\mathbb{R}^4$.
- (True/False) The set of polynomials in P_2 such that $p'''(-2) = -1$ is a subspace of P_2.

  not closed under scalar multiplication
- (True/False) If the row reduced echelon form of the m by n matrix A has no free column, then $A\mathbf{x} = \mathbf{b}$ is consistent for every $\mathbf{b}$ in $\mathbb{R}^m$.
- (True/False) The set of 3 by 3 matrices whose diagonal entries sum to zero is a subspace of $M_{3\times 3}$.
- (True/False) The dimension of $M_{4\times 4}$ is 8.

Part 2: Short Answer (15 points)

Directions: Provide a short answer for each question (no work needed). The questions here should not be computationally difficult. (In particular, no involved calculations are needed.)

Short Answer 1 (6 points)

Let $A = \begin{bmatrix} 1 & 0 & 1 & -1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$. (1.5 points each)

column space:

- Find a basis for the column space of A. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$
- Find a basis for the nullspace of A.
- Find a basis for the row space of A.
- Find a basis for the left nullspace of A.

nullspace: x_3, x_4 free

$$x_3 \begin{pmatrix} -1 \\ -1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\left\{ \begin{pmatrix} -1 \\ -1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right\}$$

row space: $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

left nullspace: same as nullspace(A^t)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

left nullspace is the zero subspace in $\mathbb{R}^3$ so it has an empty basis.

Short Answer 2 (4.5 points)

Consider the vector space $M_{3\times3}$, the space of all 3 by 3 real valued matrices. (1.5 points)

- Give an example of one invertible matrix in $M_{3\times3}$, and one noninvertible matrix in $M_{3\times3}$.
- Find a basis for $M_{3\times3}$ and determine the dimension of $M_{3\times3}$.
- The set of diagonal 3 by 3 matrices is a subspace of $M_{3\times3}$. Find a basis for the subspace of diagonal 3 by 3 matrices, and determine the dimension of this subspace.

invertible: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ noninvertible: $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Basis for $M_{3\times3}$: $\left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\}$

$\dim(M_{3\times3}) = 9$

Diagonal matrices in $M_{3\times3}$ have dimension 3 and a basis is

$\left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\}$

Short Answer 3 (4.5 points)

Consider P_3, the space of polynomials with real coefficients of degree less than or equal to 3. (1.5 points each)

- Do the polynomials $1, x, x^2, 1 + x + x^2$ form a basis for P_3? Briefly justify your answer.

  no, they cannot be a basis since they are not linearly independent ($1+x+x^2 = 1(1)+1(x)+1(x^2)$)
- Find six polynomials that span P_3.

  $1, x, x^2, x^3, 1+x, 1+x^2$
- Find three linearly independent polynomials in P_3.

  $1, x, x^2$