Math 54 Practice Quiz 5

September 25, 2019

Part 1: True/False (15 points)

Directions: For each item, circle either True or False. (1 point each)

- (True/False) The span of the vectors $(1, 2, 1)$, $(1, -1, 0)$, and $(1, 0, 0)$ has exactly three vectors in it. (has infinitely many vectors)
- (True/False) $1, x, x^2$ are linearly independent in P_{10} (the space of polynomials with real coefficients with degree less than or equal to 10).
- (True/False) The column space of the matrix $A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ is a subspace of $\mathbb{R}^2$. $\mathbb{R}^3$
- (True/False) Three linearly independent vectors in $\mathbb{R}^3$ form a basis for $\mathbb{R}^3$.
- (True/False) The dimension of P_{10} is 10. 11
- (True/False) The dimension of $M_{m\times n}$, the space of m by n matrices with real entries, is $m + n$. mn
- (True/False) The set of vectors in $\mathbb{R}^4$ satisfying $x_1 - x_2 + 2x_3 - x_4 = 1$ is a subspace. does not contain the zero vector
- (True/False) The set of polynomials in P_3 such that $p'(2) = 0$ is a subspace.
- (True/False) The set of 3 by 3 matrices with integer entries is a subspace of $M_{3\times3}$. not closed under multiplication by reals, e.g. $\frac{1}{2}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$
- (True/False) The left nullspace of an m by n matrix is a subspace of $\mathbb{R}^n$. $\mathbb{R}^m$
- (True/False) The matrices $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}$ form a basis for $M_{2\times2}$.
- (True/False) Every basis for a vector space contains the same number of elements.
- (True/False) A square matrix whose reduced row echelon form has a pivot in every column is invertible.
- (True/False) There is a 2 by 3 matrix with a row space that is all of $\mathbb{R}^3$.
- (True/False) $AB = BA$ for any two square matrices A and B of the same size.

Part 2: Short Answer (15 points)

Directions: Provide a short answer for each question (no work needed). The questions here should not be computationally difficult. (In particular, no involved calculations are needed.)

Short Answer 1 (6 points)

Let $A = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. (1.5 points each)

many possible answers

- Find three distinct vectors in the row space of A. $(0,0,0,0), (1,0,2,0), (0,1,0,1)$
- Find three distinct vectors in the nullspace of A. $x_3 \begin{pmatrix} -2 \\ 0 \\ 1 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix}$ $(0,0,0,0), (-2,0,1,0), (0,-1,0,1)$
- Find three distinct vectors in the column space of A. $(1,0,0), (0,1,0), (2,0,0)$
- Find three distinct vectors in the left nullspace of A. (This can be done easily without calculating anything.) $(0,0,1), (0,0,2), (0,0,0)$

Note: I mixed up the rows and columns by accident, so the answers given here would be correct for M2x3 instead of M3x2

Short Answer 2 (4.5 points)

many possible answers

Consider the vector space $M_{3\times 2}$, the space of all 3 by 2 real valued matrices. (1.5 points each)

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$

- Find eight vectors that span $M_{3\times 2}$, or explain why such vectors cannot exist. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$
- Find three linearly independent vectors in $M_{3\times 2}$, or explain why such vectors cannot exist.

  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$
- Find a basis for $M_{3\times 2}$. What is the dimension of $M_{3\times 2}$?

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\dim(M_{3\times 2}) = 6$

Short Answer 3 (4.5 points)

$A^t \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ -1 & -2 \end{bmatrix} \to \begin{bmatrix} 1 & 2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

Consider the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & -2 \end{bmatrix}$. (1.5 points each)

$x_2 \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

- Find a basis for the left nullspace. What is its dimension? $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ dim=1
- Find a basis for the row space. What is its dimension? $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ dim=1
- Calculate AA^t.

$$\begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & -2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ -1 & -2 \end{bmatrix} = \boxed{\begin{bmatrix} 6 & 12 \\ 12 & 24 \end{bmatrix}}$$