Math 54 Quiz 4 Study Guide

September 17, 2019

Additional Reading and Practice: See Math N54 lecture notes on matrix multiplication and invertible matrices (June 28).

Conceptual Questions

- Give an example of a surjective linear transformation from $\mathbb{R}^k$ to $\mathbb{R}^n$ where $n < k$. (Hint: restriction map)
- Give an example of an injective linear transformation from $\mathbb{R}^k$ to $\mathbb{R}^n$ where $n > k$. (Hint: inclusion map)
- Find an example of an isomorphism between $\mathbb{R}^n$ and $\mathbb{R}^n$. Show that not every linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^n$ is an isomorphism.
- Explain why any linear transformation between Euclidean spaces of different dimensions cannot be an isomorphism.

Problems

Problem 1

Let T be a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ given by

$$T(1,0) = (1,2,-1) \quad T(0,1) = (0,1,3)$$

Find a matrix A such that $T(x) = Ax$. Determine if T is injective, surjective, and an isomorphism.

Problem 2

Compute

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^{2019}$$

Problem 3

Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation that rotates by 60^o about the z axis. Find a matrix A such that $T(x) = Ax$. Is T an isomorphism?

Problem 4

Find all vectors in the range of the linear transformation T_A defined by the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 & -1 \\ 0 & 2 & 2 & 2 \\ 1 & 3 & 3 & 2 \end{bmatrix}$$

Find all vectors in the kernel of T_A. Is T an injection, a surjection, an isomorphism?

Problem 5

Suppose that $T : \mathbb{R}^3 \to \mathbb{R}^3$ is a linear transformation such that

$$T(1,1,0) = (0,1,-1) \quad T(0,2,1) = (1,0,-1) \quad T(1,1,1) = (0,-1,-3)$$

Find a matrix A such that $T(x) = Ax$. Is T an isomorphism? If it is, find a formula for the inverse linear transformation.

Problem 6

A square matrix is called **upper triangular** if $a_{ij} = 0$ if $i > j$. Show that any linear transformation defined by an upper triangular matrix with nonzero diagonal entries is an isomorphism.

Problem 7

Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

and use the inverse of A to solve the system

$$x_1 + 2x_2 + x_3 = 2$$

$$x_2 + x_3 = 1$$

$$x_1 - x_2 = 3$$

without row reducing.