Math 54 Quiz 3 Study Guide

September 10, 2019

Additional Reading and Practice: See Math N54 lecture notes on Gaussian elimination (June 26, June 27) and Math N54 Midterm 1 Solutions, Problems 1e, 1h, 4a, 4b for worked out examples

Conceptual Questions

- Explain why if $k > n$, then k vectors cannot be linearly independent in $\mathbb{R}^n$.
- Explain why if $k < n$, then k vectors cannot span $\mathbb{R}^n$.
- Explain why any n linearly independent vectors in $\mathbb{R}^n$ must be a basis for $\mathbb{R}^n$.
- Explain why any n vectors that span $\mathbb{R}^n$ must be a basis for $\mathbb{R}^n$.
- True or false: If $v_1, v_2, ..., v_k$ are linearly dependent, then one vector must be a scalar multiple of the other.
- Explain why any basis for $\mathbb{R}^n$ must have exactly n vectors. (Hint: Look at the first two conceptual questions.)

Problems

Problem 1

Show that the vectors $(1, 2, 1)$, $(-1, 0, -1)$, $(0, -1, 2)$, and $(-1, 1, 1)$ span $\mathbb{R}^3$. Write $(1, 1, 1)$ as a linear combination of these vectors.

Problem 2

Show that $(1, -1, 2, 1)$, $(2, 1, 0, 1)$, and $(0, 1, 1, 1)$ are linearly independent in $\mathbb{R}^4$.

Problem 3

Show that the vectors $(1, -1, 1)$, $(-2, 0, 1)$, and $(2, 1, -1)$ are a basis for $\mathbb{R}^3$.

Problem 4

Characterize all vectors (a, b, c, d) in $\text{Span}\{(1, 2, 1, -1), (-1, 0, 2, 1), (-1, 2, 5, 1)\}$. Give an example of a vector in $\mathbb{R}^4$ that is not in this span.

Problem 5

Show that if $v_1, v_2, ..., v_k$ is linearly dependent, then for any vector w, the set of vectors $v_1, v_2, ..., v_k, w$ is also linearly dependent.

Problem 6

Show that if $v_1, v_2, ..., v_k$ span $\mathbb{R}^n$, then so do $v_1, v_2, ..., v_k, w$ where w is any vector in $\mathbb{R}^n$.

Problem 7

Show however that if $v_1, v_2, ..., v_n$ are a basis for $\mathbb{R}^n$, then if w is any vector in $\mathbb{R}^n$, the set $\{v_1, v_2, ..., v_n, w\}$ is never a basis for $\mathbb{R}^n$.

Problem 8

Extend the linearly independent set $\{(1, 2, 1), (1, 0, 1)\}$ to a basis for $\mathbb{R}^3$.