Math 54 Quiz 3

September 12, 2019

Question 1 (3 points)

Directions: For each item, circle either True or False. (0.5 points each)

- (True/False) There exist a set of 2019 vectors total that altogether span $\mathbb{R}^3$.
- (True/False) The three vectors $(1,2,1,3,4), (1,2,0,0,1), (-1,2,1,-1,-1)$ form a basis for $\mathbb{R}^5$.
- (True/False) Three linearly independent vectors in $\mathbb{R}^3$ must also span $\mathbb{R}^3$.
- (True/False) Any set of vectors that contains the zero vector is linearly dependent.
- (True/False) If v_1, v_2, v_3, v_4 are linearly independent in $\mathbb{R}^4$, then v_1, v_2, v_3 are also linearly independent in $\mathbb{R}^4$.
- (True/False) If v_1, v_2, v_3 are linearly dependent in $\mathbb{R}^3$, then v_1, v_2 are also linearly dependent in $\mathbb{R}^3$.

Question 2 (6 points)

Determine if $(1,2,3,1)$, $(0,1,1,1)$, $(0,1,2,-1)$, and $(-1,-3,-5,0)$ form a basis for $\mathbb{R}^4$.

Check linear independence.

$$c_1(1,2,3,1) + c_2(0,1,1,1) + c_3(0,1,2,-1) + c_4(-1,-3,-5,0) = (0,0,0,0)$$

$$\begin{cases} c_1 - c_4 = 0 \\ 2c_1 + c_2 + c_3 - 3c_4 = 0 \\ 3c_1 + c_2 + 2c_3 - 5c_4 = 0 \\ c_1 + c_2 - c_3 = 0 \end{cases}$$

$$A = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 2 & 1 & 1 & -3 \\ 3 & 1 & 2 & -5 \\ 1 & 1 & -1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & 1 & 2 & -2 \\ 0 & 1 & -1 & 1 \end{bmatrix} \begin{matrix} \\ R_2 - 2R_1 \\ R_3 - 3R_1 \\ R_4 - R_1 \end{matrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -2 & 2 \end{bmatrix} \begin{matrix} \\ \\ R_3 - R_2 \\ R_4 - R_2 \end{matrix}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \\ R_2 - R_3 \\ \\ R_4 + 2R_3 \end{matrix}$$

not just trivial soln $\Rightarrow$ so not linearly independent

So not a basis for $\mathbb{R}^4$.

Question 3 (6 points)

Find the span of the vectors $(1, 2, 1, 1)$, $(1, 0, 0, 1)$, $(2, 2, 1, 2)$ in $\mathbb{R}^4$. (In particular, describe all (a, b, c, d) that are in this span by giving conditions on a, b, c, and d).

Find one vector in $\mathbb{R}^4$ that is not in $\text{Span}\{(1, 2, 1, 1), (1, 0, 0, 1), (2, 2, 1, 2)\}$.

(a,b,c,d) is in the span if there is a solution (c_1, c_2, c_3) to

$$c_1(1,2,1,1) + c_2(1,0,0,1) + c_3(2,2,1,2) = (a,b,c,d)$$

$$\begin{cases} c_1 + c_2 + 2c_3 = a \\ 2c_1 \qquad + 2c_3 = b \\ c_1 \qquad + c_3 = c \\ c_1 + c_2 + 2c_3 = d \end{cases}$$

Find (a,b,c,d) for which this system is consistent.

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & a \\ 2 & 0 & 2 & b \\ 1 & 0 & 1 & c \\ 1 & 1 & 2 & d \end{array}\right] \to \begin{array}{l} \left[\begin{array}{ccc|c} 1 & 1 & 2 & a \\ 0 & 0 & 0 & b-2c \\ 1 & 0 & 1 & c \\ 0 & 0 & 0 & -a+d \end{array}\right] \end{array} \begin{array}{l} \\ R_2 - 2R_3 \\ \\ R_4 - R_1 \end{array}$$

$$\to \left[\begin{array}{ccc|c} 1 & 1 & 2 & a \\ 0 & 1 & 1 & a-c \\ 0 & 0 & 0 & b-2c \\ 0 & 0 & 0 & -a+d \end{array}\right] \begin{array}{l} \\ R_1 - R_3 \\ \\ \\ \end{array} \to \left[\begin{array}{ccc|c} 1 & 0 & 1 & c \\ 0 & 1 & 1 & a-c \\ 0 & 0 & 0 & b-2c \\ 0 & 0 & 0 & -a+d \end{array}\right] \begin{array}{l} R_1 - R_2 \\ \\ \\ \\ \end{array}$$

$b - 2c = 0$ and $-a + d = 0$

$$\boxed{a = d \text{ and } b = 2c}$$

$$\boxed{\begin{array}{l} (1, 0, 0, -1) \text{ is not in the span} \\ \text{since } 1 \neq -1 \end{array}}$$