Math 54 Quiz 2 Study Guide

September 2, 2019

Additional Reading and Practice: See Math N54 lecture notes on Gaussian elimination (June 25) and Math N54 Problem Set 1 Solutions for worked out examples

Conceptual Questions

- Show that a homogeneous underdetermined system (a system that has more variables than equations) always has infinitely many solutions.
- Is every homogeneous system consistent?
- Show by example that an overdetermined system (a system that has more equations than variables) can be either consistent or inconsistent.
- True or false: If the reduced row echelon form of the augmented matrix for a system has a row of zeros, the system has infinitely many solutions.

Problems

Problem 1

Write the solutions to the following system (if consistent) using parametric vector form.

$$x_1 - x_2 - x_3 = 2$$

$$2x_1 - 5x_2 + 3x_3 = 1$$

$$4x_1 - 7x_2 + x_3 = 0$$

$$x_1 + 4x_2 = 0$$

$$-2x_1 - x_2 = 0$$

$$3x_1 + 2x_2 = 0$$

$$x_1 + 3x_2 + x_3 = 2$$
$$x_1 - x_2 - x_3 = 0$$
$$2x_2 + x_3 = 3$$

$$x_1 + x_2 + 2x_3 + x_4 = 2$$
$$x_1 - 2x_2 + 2x_3 - x_4 = 1$$

Problem 2

Find conditions on a, b, and c so that the following system is consistent.

$$x_1 - x_2 + x_3 = a$$
$$2x_1 - x_2 + 3x_3 = b$$
$$4x_1 - 3x_2 + 5x_3 = c$$

Problem 3

For what values of h does the following system have a unique solution? (Note this system is always consistent since it is homogeneous, but it might not necessarily have a unique solution.)

$$x_1 + 2x_2 + hx_3 = 0$$
$$2x_1 + 3x_2 - x_3 = 0$$
$$4x_1 + 7x_2 + 3x_3 = 0$$

Problem 4

Parametrize all points in \mathbb{R}^3 that satisfy $x_1 + 2x_2 - 3x_3 = 1$. (Hint: Consider this as a linear system of three variables and just one equation, and write the solution in parametric vector form.)