Math 1A: Discussion 9/7/2018 Solutions

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Problem Set 1

Question 1

We have that $\log_3 27$ is asking for the exponent we need to raise 3 to, to get 27. We note that $3^3 = 27$, so

$$\log_3 27 = 3$$

Next, we note that $e^{-1/2} = \frac{1}{\sqrt{e}}$, so

$$\ln\left(\frac{1}{\sqrt{e}}\right) = -\frac{1}{2}$$

For the next problem, we want to write $\frac{2}{\sqrt[5]{4}}$ as 4 raised to some power. We have that

$$\frac{2}{\sqrt[5]{4}} = \frac{2}{(2^2)^{1/5}} = \frac{2}{2^{2/5}} = 2^{3/5} = (4^{1/2})^{3/5} = 4^{3/10}$$

so that

$$\log_4\left(\frac{2}{\sqrt[5]{4}}\right) = \frac{3}{10}$$

Finally, $\log_5(3)$ is something that we cannot compute directly. But we know it's a number where if we take 5 and raise 5 to that number, we will get 3. But that's exactly what this question is doing. So

$$5^{\log_5(3)} = 3$$

Question 2

We cannot take a logarithm of a nonpositive number. So to find the domain, we want to find where

$$x^2 - 3x + 2 > 0$$

We factor this as

$$(x-1)(x-2) > 0$$

To solve this we find the roots, of $(x-1)(x-2)$ which are $x = 1$ and $x = 2$. Drawing a number line and plotting the zeros of this function on the line, the number line gets separated into three regions: $x < 1$, $1 < x < 2$, and $x > 2$. Trying a point from each region we see that

- $f(x) = (x-1)(x-2)$ is positive for $x < 1$.
- $f(x) = (x-1)(x-2)$ is negative for $1 < x < 2$.
- $f(x) = (x-1)(x-2)$ is positive for $x > 2$.

So we see that $(x-1)(x-2) > 0$ when $x < 1$ or $x > 2$. So the domain is $x < 1$ or $x > 2$, also written as $(-\infty, 1) \cup (2, \infty)$ using interval notation ($\cup$, the union, represents combining the two sets together).

Problem Set 2

Question 3

We have the function $f(t) = t^2 + 4t$ for $0 \leq t \leq 5$. The average velocity on $[2, 3]$ is

$$\frac{f(3) - f(2)}{3 - 2} = \frac{21 - 12}{1} = 9$$

The average velocity on $[2, 2.5]$ is

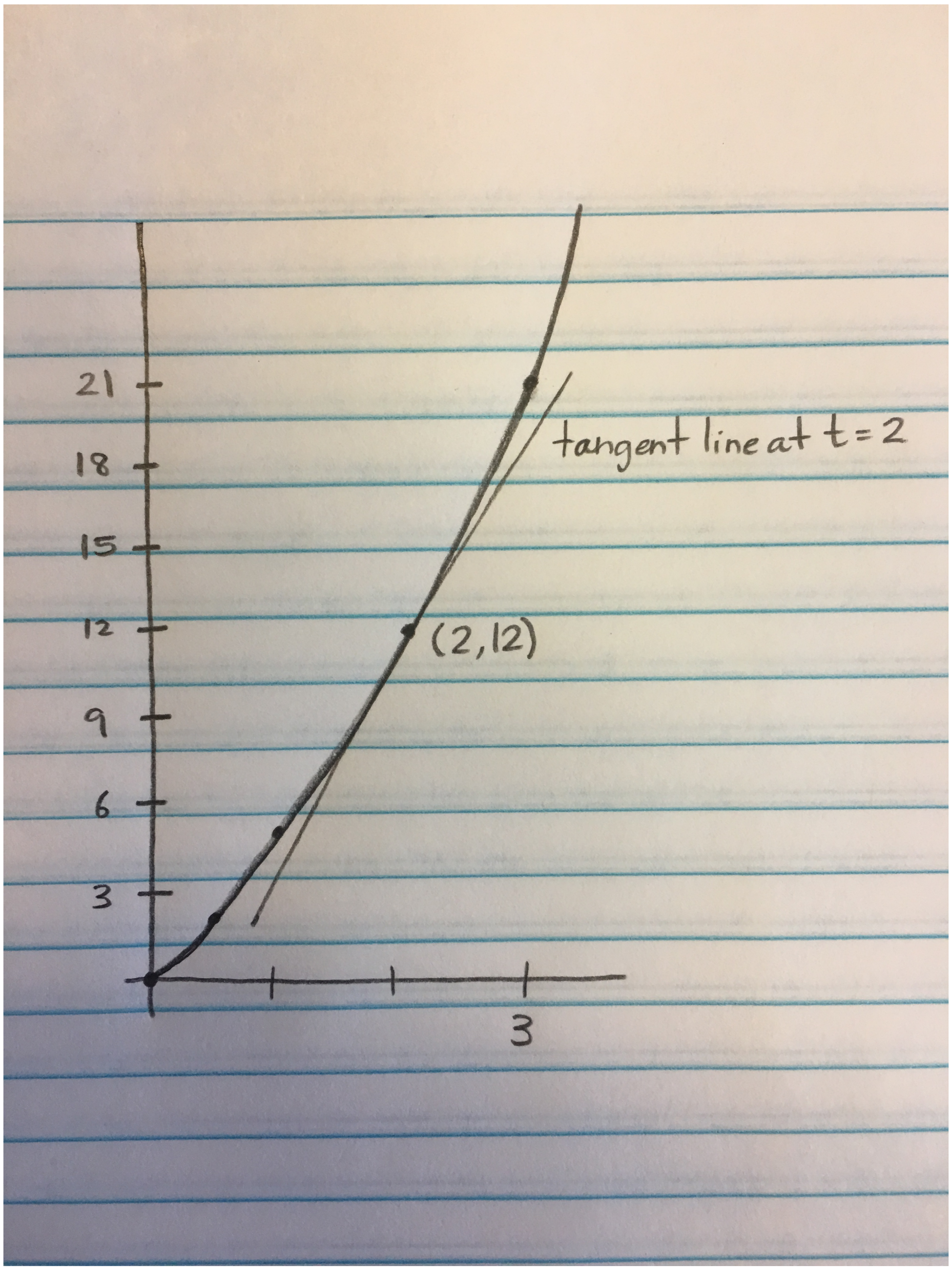
$$\frac{f(2.5) - f(2)}{2.5 - 2} = 8.5$$

The average velocity on $[2, 2.1]$ is

$$\frac{f(2.1) - f(2)}{2.1 - 2} = 8.1$$

So we estimate that the instantaneous velocity at $t = 2$ is 8.

We draw a graph below. The average velocity on $[a, b]$ is just the slope of the secant line that connects the points $(a, f(a))$ and $(b, f(b))$, while the instantaneous velocity at $t = 2$ is the slope of the tangent line to the graph at $t = 2$. Note that the approximations above are slopes of secant lines and as the time interval is getting smaller and smaller, this slope is approaching the actual slope of the tangent line.


21
18
15
12
9
6
3
3
tangent line at t=2
(2,12)