Math 1A: Discussion 9/5/2018 Solutions

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Problem Set 1

Question 1

$$16^{3/4} = (16^{1/4})^3 = 2^3 = 8$$

$$\frac{a^3b^5}{a^4b^2} = a^{3-4}b^{5-2} = a^{-1}b^3 = \frac{b^3}{a}$$

$$(x^2)^{-2}x^3 = x^{-4}x^3 = x^{-1}$$

$$(x^2yz)^{-1} \cdot (2xz)^2 = x^{-2}y^{-1}z^{-1} \cdot 4x^2z^2 = 4x^{-2+2}y^{-1}z^{-1+2} = 4x^0y^{-1}z^1 = \frac{4z}{y}$$

$$\frac{(2x)^2y^3}{(4x)^{3/2}} = \frac{4x^2y^3}{8x^{3/2}} = \frac{1}{2}x^{2-3/2}y^3 = \frac{1}{2}x^{1/2}y^3$$

Question 2

To solve $2^{x+2} = \frac{1}{4}$, we note that $2^{-2} = \frac{1}{4}$, so that we are solving

$$x + 2 = -2$$

So $x = -4$.

Next, to solve $\left(\frac{1}{2}\right)^{2-x} = \frac{1}{16}$, we note that $\left(\frac{1}{2}\right)^4 = \frac{1}{16}$, so we are solving

$$2 - x = 4$$

So $x = -2$.

Finally, to solve $3^{(x^2)} = \frac{1}{27}$, we note that $3^{-3} = \frac{1}{27}$, so we are solving $x^2 = -3$. But this has no solutions since x^2 is nonnegative for all x. So this equation has no solution.

Question 3

For $(3, 1)$ and $(5, 4)$, we have that

$$1 = Ar^3$$

$$4 = Ar^5$$

Dividing, we have that

$$r^2 = \frac{Ar^5}{Ar^3} = \frac{4}{1} = 4$$

Since r is nonnegative, we have that $r = 2$. Then, $1 = A(2)^3$, so $A = 1/8$. So the equation is

$$y = \frac{1}{8}2^x$$

For $\left(-2, \frac{1}{2}\right)$ and $\left(2, \frac{1}{32}\right)$, we have that

$$\frac{1}{32} = Ar^2$$

$$\frac{1}{2} = Ar^{-2}$$

Dividing, we have that

$$r^4 = \frac{Ar^2}{Ar^{-2}} = \frac{1/32}{1/2} = \frac{1}{16}$$

So since r is nonnegative, we have that $r = 1/2$. Using the first equation, $1/32 = A(1/2)^2$, so that $A = 1/8$. So we have that

$$y = \frac{1}{8}\left(\frac{1}{2}\right)^x$$

Finally, for $\left(2, -\frac{1}{3}\right)$ and $(5, -9)$, we have that

$$-\frac{1}{3} = Ar^2$$

$$-9 = Ar^5$$

Dividing, we have that

$$r^3 = \frac{Ar^5}{Ar^2} = \frac{-9}{-1/3} = 27$$

So $r = 3$. Using the first equation, $-1/3 = A(3)^2$, so that $A = -1/27$. So the equation is

$$y = -\frac{1}{27} \cdot 3^x$$

Problem Set 2

Question 4

For the first part, we have that

$$(f \circ g)(x) = f(g(x)) = \frac{3}{e^x - 1}$$

The domain of f is $x \neq 1$. So to find the domain of the composite $f \circ g$ we look for values of x with $g(x) = 1$. The only such value where $e^x = 1$ is $x = 0$. So the domain is $x \neq 0$, which can also be expressed as $(-\infty, 0) \cup (0, \infty)$.

For the second part, the composite function is

$$(f \circ g)(x) = \sqrt{2(x+1) - 3} = \sqrt{2x - 1}$$

The domain of f is $x \geq 3/2$. So we want to find values of x for which $g(x) \geq 3/2$. We can easily see that $x + 1 \geq 3/2$ when $x \geq 1/2$. So the domain is $x \geq 1/2$, which can also be expressed as $[1/2, \infty)$ (which can also be seen directly from the expression for $f \circ g$).

For the last part, we have that the composite function is

$$(f \circ g)(x) = \frac{1}{2\left(2^{|x|}\right)^2 - 5(2^{|x|}) + 2} = \frac{1}{2(2^{2|x|}) - 5(2^{|x|}) + 2}$$

Factoring the denominator of f as $(2x-1)(x-2)$, we see that the domain of f is $x \neq 1/2$, $x \neq 2$. So we are solving $2^{|x|} = 1/2$ and $2^{|x|} = 2$.

We note that $2^{-1} = 1/2$, so since $|x|$ can never be equal to -1, we have that $2^{|x|} = 1/2$ has no solutions. To solve $2^{|x|} = 2$, we note that $2^1 = 2$, so we have that the solutions are $x = -1, 1$.

So the domain of $f \circ g$ is $x \neq -1, 1$, which can also be expressed as $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$.

Question 5

For the graphs of these functions, see the images on the website.

From the graph of the function f, we see that f passes the Horizontal Line Test. To find the inverse of f, reverse x and y and solve for y.

$$x = 2y^3 + 3$$
$$x - 3 = 2y^3$$
$$y^3 = \frac{x-3}{2}$$
$$y = f^{-1}(x) = \sqrt[3]{\frac{x-3}{2}}$$

From the graph of the function g, we find that g passes the Horizontal Line Test. To find the inverse of g, reverse x and y and solve for y.

$$x = -\frac{1}{2}(y-1)^5 + 2$$
$$x - 2 = -\frac{1}{2}(y-1)^5$$
$$-2(x-2) = (y-1)^5$$
$$y - 1 = \sqrt[5]{-2(x-2)}$$
$$y = 1 + \sqrt[5]{-2x+4}$$

Finally, h does not have an inverse because it does not pass the Horizontal Line Test.

Question 6

We note that

$$f(1) = \frac{1}{3}(1-1)^7 + 2 = 2$$

so that $f^{-1}(2) = 1$.

To find an expression for f^{-1}, we switch x and y and solve for y.

$$x = \frac{1}{3}(y-1)^7 + 2$$

$$\frac{1}{3}(y-1)^7 = x-2$$
$$(y-1)^7 = 3x-6$$
$$y-1 = \sqrt[7]{3x-6}$$
$$f^{-1}(x) = y = 1+\sqrt[7]{3x-6}$$

Next, we consider

$$f(x) = \frac{x+3}{2x+1}$$

To find $f^{-1}(2)$, we solve the equation

$$\frac{x+3}{2x+1} = 2$$
$$x+3 = 4x+2$$
$$3x = 1$$
$$x = 1/3$$

So we have that $f^{-1}(2) = 1/3$.

To find f^{-1}, we switch x and y and solve for y.

$$x = \frac{y+3}{2y+1}$$
$$2xy + x = y+3$$

This is harder to solve for y. Move everything with a y in it to one side and factor out y to isolate it.

$$2xy - y = 3-x$$
$$y(2x-1) = 3-x$$

Dividing both sides by $2x-1$, we have that

$$f^{-1}(x) = y = \frac{3-x}{2x-1}$$

Problem Set 3

Question 7

See images on website.